Pre-Calculus
Review - $1^{\text {st }}$ Semester Exam

Name:
Period:

- The final exam covers Ch. P, 1, 2, 7, 3, 9 and will account for $15 \%$ of your semester grade.
- Questions marked with ** are calculator OK.


## Chapter P

(\#1-2) Solve the inequality and draw a number line graph of the solution set.

1. $-1 \leq 3 x-2<7$
2. $4(1-x)+5(1+x)>3 x-1$
(\#3-4) Write an equation that fits the line described below in slope-intercept form.
3. The line through the points $(-4,5)$ and $(4,3)$
4. The line through the points $(4,2)$ and $(-3,1)$
(\#5-8) Solve using the method indicated.
5. Quadratic formula: $2 x^{2}-3 x+1=0$
6. Factoring: $x^{2}-x-20=0$
7. Graphically: $4 x^{2}-8 x+3=0 * *$
8. Identify the domain of: $f(x)=\frac{x-2}{\sqrt{x+5}}$
(\#9-13) Solve algebraically and express your answer in interval notation:
9. $|x+4| \geq 5$
10. $|x-3|<2$
11. $|4-3 x|-2<4$
12. $x^{3}-x \geq 0$
13. $\frac{x+2}{2 x-3} \geq 0$

## Chapter 1

1. Use your calculator to find all local maxima and minima and the values of x where they occur. Round values to 2 decimals places. $h(x)=-x^{3}+2 x-3^{* *}$
2. Graph the function and identify intervals on which the function is increasing, decreasing, or constant. Remember, your intervals are based on the " x " values. $f(x)=x^{3}-x^{2}-2 x^{* *}$
3. State whether the function is odd, even, or neither. Support graphically. $f(x)=2 x^{4 * *}$
4. State whether the function is odd, even, or neither. Support graphically. $y=2 x^{3}-3 x^{* *}$
5. Use a method of your choice to find all horizontal and vertical asymptotes. (You should be able to do it both graphically and algebraically.) $g(x)=\frac{x+2}{3-x}$ and $f(x)=\frac{x^{2}+2}{x^{2}-1}$
6. Find $(f \circ g)(3)$ and $(g \circ f)(-2)$ if $f(x)=2 x-3$ and $g(x)=x+1$
7. Find $f(g(x))$ and $g(f(x))$ if $f(x)=3 x+2$ and $g(x)=x-1$
8. List, in order, the transformations applied to $y=x^{2}$ to obtain the graph of $y=(x-1)^{2}+3$.
(\#9-10) Describe a basic graph (original) and a sequence of transformations that can be used to produce a graph of the given function.
9. $y=2(x-3)^{2}-4$
10. $y=(3 x)^{2}-4$
(\#11-12) write the equation of the new function based on the given transformations applied to the given function.
11. $y=x^{2}$; Vertical stretch by a factor of 3 ; shift right 4 units.
12. $y=|x|$; shift left 2 units; vertical stretch by factor of 2; shift down 4 units
(\#13-14) Find the domain and range of the function. You should be able to do so algebraically and graphically.
13. $h(x)=(x-2)^{2}+5$
14. $k(x)=\sqrt{4-x^{2}}-2$
15. Find all discontinuities for the function: $f(x)=\frac{5 x-25}{x^{2}-5 x}$. Then, graph the function and describe the behavior around the vertical asymptote.
16. State all intervals on which the function is increasing. (Remember, the intervals are based off the " $x$ " values of the ordered pairs.) $y=\frac{x^{2}-1}{x^{2}-4} * *$
(\#17-18) Find the equation for $f^{-1}(x)$.
17. $f(x)=2 x+3$
18. $f(x)=\sqrt[3]{x-8}$
(\#19-20) Let $f(x)=\sqrt{x}$ and $g(x)=x^{2}-4$.
19. Find an expression for $(f \circ g)(x)$ and give its domain.
20. Find an expression for $(g \circ f)(x)$ and give its domain.
21. Describe the end behavior of the function $f(x)=x$ using limit notation.

## Chapter 2

1. Sketch a graph of the function by hand. $f(x)=x^{2}-4 x+6$
2. Describe the end behavior of the following graphs. (Use correct limit notation)
a. $f(x)=3 x^{4}-5 x^{2}+3$
b. $f(x)=-x^{3}+7 x^{2}-4 x+3$
3. Divide $\mathrm{f}(\mathrm{x})$ by $\mathrm{d}(\mathrm{x})$.
$f(x)=x^{3}+4 x^{2}+7 x-9 ; d(x)=x+3$
4. Divide using synthetic division.

$$
\frac{x^{3}-5 x^{2}+3 x-2}{x+1}
$$

5. Use the remainder theorem to find the remainder when $f(x)=2 x^{2}-3 x+1$ is divided by $x-2$.
6. Use the remainder theorem to find the remainder when $f(x)=2 x^{3}-3 x^{2}+4 x-7$ is divided by $x-2$.
7. Write a polynomial in FACTORED form with a degree of 4 with zeros at $-1,3$ and $1-2 \mathrm{i}$.
8. Use the factor theorem to determine whether $x-1$ is a factor of $x^{3}-x^{2}+x-1$.
9. Use the factor theorem to determine whether $\mathrm{x}-2$ is a factor of $x^{3}+3 x-4$.
10. Factor the following polynomial: $f(x)=2 x^{4}+7 x^{3}-7 x^{2}-35 x-15^{* *}$
11. Find all zeros - both real and imaginary - of the following polynomial: $f(x)=x^{4}-2 x^{3}+x^{2}-8 x-12 * *$
12. See textbook page 215\#17, 19 (Section 2.5 - matching polynomial graphs)
13. Find the asymptotes and intercepts of the function. $f(x)=\frac{2}{x-3}$
14. Find the asymptotes and intercepts of the function. $f(x)=\frac{x-2}{x^{2}-2 x-3}$
15. Find the asymptotes and intercepts of the function. $f(x)=\frac{2 x^{2}+x-2}{x^{2}-1}$
16. Find the asymptotes and intercepts of the function. $f(x)=\frac{x^{2}-2 x+3}{x+2}$
17. Solve the equation algebraically. Check for extraneous solutions! $\frac{3 x}{x+5}+\frac{1}{x-2}=\frac{7}{x^{2}+3 x-10}$
18. Determine the values of x that cause the polynomial function to be a) zero, b) positive, and c) negative $f(x)=(x+2)(x+1)(x-5)$
19. Complete a sign chart and solve the polynomial. Express your answer in interval notation.

$$
(x+1)(x-3)^{2}>0
$$

20. Solve the polynomial $x^{3}-x^{2}-2 x \geq 0$ graphically. Express your answer in interval notation. ${ }^{* *}$
21. Solve the inequality using a sign chart. Support your answer graphically. Express your answer in interval notation. $\frac{x-1}{x^{2}-4}<0$
22. Designing a cardboard box. See textbook page 195 \#67. **

## Chapter 7 (Matrices)

1. Use the matrices $\mathrm{A}, \mathrm{B}, \mathrm{C}$, and D to find the following:
$A=\left[\begin{array}{cc}3 & -2 \\ 4 & 2\end{array}\right]$
$B=\left[\begin{array}{cc}4 & -2 \\ -6 & 3\end{array}\right]$
$C=\left[\begin{array}{cc}-1 & 0 \\ 3 & 5 \\ 4 & -2\end{array}\right]$
$D=\left[\begin{array}{ccc}6 & 3 & -8 \\ 4 & -2 & -1 \\ 5 & 3 & 1\end{array}\right]$
a. $2 A-3 B$
b. $C \times B$
c. $A \times C$
d. $D \times C$
e. $B^{-1}$
f. The determinant of $A$
2. Solve the system using the indicated method: $2 x-3 y+z=14 \quad{ }^{* *}$

$$
3 x-4 y+6 z=29
$$

$$
x+2 y-5 z=-11
$$

a. Inverse Matrices
b. Reduced Row Echelon Form

## Chapter 3.1-3.5 (Exponentials, Logarithms, and Logistics)

1. Find the equation of an exponential that goes through the points $(0,3)$ and $(2,12)$.
2. The population of a town is decreasing at a rate of $1.5 \%$ per year. The initial population was 2000 people. When will the population reach 1000 if the population continues to decay at this rate? **
3. Write the equation of a logistic function that has an initial value of 16 , limit to growth of 128 , and passes through the point $(5,32)$. ${ }^{* *}$
(\#4-6) Solve the log expression.
4. $\log _{4} \frac{1}{16}$
5. $\ln e^{-5}$
6. $\log _{2} 64$
(\#7-9) Solve the equation for $x$.
7. $4 \log x=8$
8. $\ln (x-3)+\ln (x+4)=3 \ln 2$
9. $\log _{3} 3 x+\log _{3} x=4$
10. Solve the equation for $\mathrm{x}: 4000+200(2.08)^{x}=6000^{* *}$
11. Graph the following logarithmic function: $f(x)=\log _{2} x$

## Chapter 9 (Combinatorics, Permutations and Probability)

1. At a Wendy's restaurant a customer can order a single, double or triple hamburger with up to 12 condiments (ketchup, mustard, onions, pickles, etc.). How many different ways can the customer order a burger?**
2. Write the sample space for all outcomes of flipping 3 different coins.
3. How many distinguishable 9 letter "words" can be formed using the letters in TENNESSEE?**
4. How many ways can 9 starters be chosen from a team of 20 baseball players if each has equal abilities?**
5. 17 baseball players show up to a game. How many ways can the manager write out a batting order of 9 batters?**
6. In your class there are 9 girls and 11 boys. Your teacher will be selecting a group of 7 students to compete in a mathematics contest.
a. In how many ways can she select the group of 7? **
b. How many ways can she select a group with exactly 5 boys? **
c. How many ways can she select no more than 2 girls? **
7. Assume that the probability of giving birth to a boy or a girl is the same. **
a. In a family of 3 children, what is the probability of:
i. Having only one girl?
ii. Having either all boys or all girls?
b. In a family of 4 children, what is the probability of:
i. Having three girls and one boy?
ii. Having at least three girls?
iii. Having 2 of each?
8. The probability of a colored gum ball coming out of a machine is shown in the table below.**

| RED | WHITE | BLUE | GREEN | YELLOW |
| :---: | :---: | :---: | :---: | :---: |
| 0.25 | 0.33 | 0.18 | 0.16 | 0.08 |

If one gumball is purchased, tind each probability:
a. P (White)
b. $\mathrm{P}($ Blue or Green $)$
c. P (Not Yellow)

If two gumballs are purchased, find each probability:
a. P(both Red)
b. $\mathrm{P}($ Green then Yel.)
c. P(Blue and Green)
d. P (No White)

