

1. The half-life of a substance is 4 minutes. The original mass is 100 grams. How much of the substance remains after 15 minutes?

$$A(t) = 100 \left(\frac{1}{2}\right)^{t/4}$$

$$A(t) = 100 \left(\frac{1}{2}\right)^{15/4} \approx \boxed{7.43 \text{ grams}}$$

2. Write a logistic function if:

Initial value = 10, Limit to growth = 50, Passes through (2,30)

$$(0, 10)$$

$$C = 50$$

$$x \ y$$

$$10 = \frac{50}{1 + a \cdot b^0}$$

$$30 = \frac{50}{1 + 4b^2}$$

$$10 + 10a = 50$$

$$10a = 40$$

$$a = 4$$

$$30 + 120b^2 = 50$$

$$120b^2 = 20$$

$$b^2 = \frac{1}{6}$$

$$b \approx .41$$

$$f(x) = \frac{50}{1 + 4(.41)^x}$$

Solve for x:

$$3. \log(x+1) - \log(2x-1) = \frac{1}{2} \log 4 \rightarrow \sqrt{4} = 2$$

$$\log \frac{x+1}{2x-1} = \log 4^{1/2}$$

$$\frac{x+1}{2x-1} = 2$$

$$4x - 2 = x + 1$$

$$5. 1 + 2e^{-2x} = 6 \quad \boxed{x=1}$$

$$2e^{-2x} = 5$$

$$e^{-2x} = 2.5$$

$$\ln 2.5 = -2x$$

$$x = \frac{\ln 2.5}{-2} \approx \boxed{-.458}$$

$$4. \log_4(x-3) = -1$$

$$4^{-1} = x - 3$$

$$\frac{1}{4} = x - 3$$

$$\boxed{3\frac{1}{4} = x}$$

$$6. \left(\frac{1}{4}\right)^{4x-1} = 8^{3x}$$

$$\left(\frac{1}{2}\right)^{2(4x-1)} = \left(\frac{1}{2}\right)^{-3(3x)}$$

$$2(4x-1) = -3(3x)$$

$$8x - 2 = -9x$$

$$17x = 2 \quad \boxed{x = 2/17}$$

7. Write the equation of an exponential function that goes through (0,2) and (3,12). (Round b-value to nearest hundredths.)

$$y = a \cdot b^x$$

$$12 = 2 \cdot b^3$$

$$6 = b^3$$

$$1.82 \approx b$$

$$\boxed{y = 2 \cdot 1.82^x}$$

8. The population of Chicago has been increasing at 1.2% per year. If the population is 3,000,000 in 2000, when will the population reach 3,500,000? growth  $r = .012$   $P_0$

$$P(t) = 3,000,000 (1 + .012)^t$$

$$3,500,000 = 3,000,000 (1.012)^t$$

$$\frac{35}{30} = (1.012)^t$$

$$t = \frac{\log \frac{35}{30}}{\log 1.012} \approx \boxed{12.923 \text{ years}}$$