Chapter 3 Review Exercises
(pp. 150–153)

1. $\frac{dy}{dx}=\frac{d}{dx}\left(x^5-\frac{1}{8}x^2+\frac{1}{4}x\right)=5x^4-\frac{1}{4}x+\frac{1}{4}$

2. $\frac{dy}{dx}=\frac{d}{dx}(3-7x^3+3x^7)=-21x^2+21x^6$

3. $\frac{dy}{dx}=\frac{d}{dx}(2\sin x\cos x)$
$=2(\sin x)\frac{d}{dx}(\cos x)+2(\cos x)\frac{d}{dx}(\sin x)$
$=-2\sin^2 x+2\cos^2 x$
$=2\cos 2x$

4. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{2x+1}{2x-1}\right)$
$=\frac{(2x-1)(2)-(2x+1)(2)}{(2x-1)^2}$
$=-\frac{4}{(2x-1)^2}$

5. $\frac{ds}{dt}=\frac{d}{dt}[(t^2-1)(t^2+1)]$
$=\frac{d}{dt}[t^4-1]$
$=4t^3$

6. $\frac{ds}{dt}=\frac{d}{dt}\left(\frac{t^2+1}{1-t^2}\right)$
$=\frac{(1-t^2)(2t)-(t^2+1)(-2t)}{(1-t^2)^2}$
$=\frac{4t}{(1-t^2)^2}$

7. $\frac{dy}{dx}=\frac{d}{dx}\left(\sqrt{x}+1+\frac{1}{\sqrt{x}}\right)$
$=\frac{d}{dx}(x^{1/2}+1+x^{-1/2})$
$=\frac{1}{2}x^{-1/2}-\frac{1}{2}x^{-3/2}$
$=\frac{1}{2\sqrt{x}}-\frac{1}{2x^{3/2}}$

8. $\frac{dy}{dx}=\frac{d}{dx}[(x^5+1)(3x^2-x)]$
$=5x^4(3x^2-x)+(6x-1)(x^5+1)$

9. $\frac{dr}{d\theta}=\frac{d}{d\theta}(5\theta^2\sec\theta)$
$=10\theta\sec\theta+5\theta^2\sec\theta\tan\theta$

10. $\frac{dr}{d\theta}=\frac{d}{d\theta}\left(\frac{\tan\theta}{\theta^3+\theta+1}\right)$
$=\frac{\sec^2\theta(\theta^3+\theta+1)-\tan\theta(3\theta^2+1)}{(\theta^3+\theta+1)^2}$

11. $\frac{dy}{dx}=\frac{d}{dx}(x^2\sin x+x\cos x)$
$=x^2\cos x+2x\sin x+x(-\sin x)+\cos x$
$=(x^2+1)\cos x+x\sin x$

12. $\frac{dy}{dx}=\frac{d}{dx}(x^2\sin x-x\cos x)$
$=x^2\cos x+2x\sin x-[x(-\sin x)+\cos x]$
$=(x^2-1)\cos x+3x\sin x$

13. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{\tan x}{2x^3}\right)$
$=\frac{2x^3\sec^2 x-6x^2\tan x}{4x^6}$
$=\frac{x\sec^2 x-3\tan x}{2x^4}$

14. $\frac{dy}{dx}=\frac{d}{dx}(\tan x-\cot x)$
$=\sec^2 x-(-\csc^2 x)$
$=\sec^2 x+\csc^2 x$

15. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{1}{\sin x+\cos x}\right)$
$=\frac{(\sin x+\cos x)\cdot 0-1(\cos x-\sin x)}{(\sin x+\cos x)^2}$
$=\frac{\sin x-\cos x}{(\sin x+\cos x)^2}$

16. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{1}{\sin x}+\frac{1}{\cos x}\right)$
$=\frac{d}{dx}(\csc x+\sec x)$
$=-\csc x\cot x+\sec x\tan x$
$=\sec x\tan x-\csc x\cot x$

17. $\frac{dV}{dr}=\frac{d}{dr}\left(\frac{4}{3}\pi r^3+8\pi r^2\right)=4\pi r^2+16\pi r$

18. $\frac{dA}{ds}=\frac{d}{ds}\left(\frac{\sqrt{3}}{4}s^2+\frac{3\pi}{4}s^2\right)$
$=\frac{\sqrt{3}}{2}s+\frac{3\pi}{4}s$
$=\left(\frac{\sqrt{3}}{2}+\frac{3\pi}{4}\right)s$

19. $\frac{ds}{dt} = \frac{d}{dt}\left(\frac{1+\sin t}{1+\tan t}\right)$
$= \frac{\cos t(1+\tan t) - \sec^2 t(1+\sin t)}{(1+\tan t)^2}$

20. $\frac{ds}{dt} = \frac{d}{dt}\left(\frac{1+\sin t}{1+\cos t}\right)$
$= \frac{\cos t(1+\cos t) - (-\sin t)(1+\sin t)}{(1+\cos t)^2}$
$= \frac{\cos t + \cos^2 t + \sin t + \sin^2 t}{(1+\cos t)^2}$
$= \frac{\cos t + \sin t + 1}{(1+\cos t)^2}$

21. $\frac{ds}{dt} = \frac{d}{dt}\left(\frac{t^{-1}+t^{-2}}{t^{-3}}\right) = \frac{d}{dt}(t^2+t) = 2t+1$

22. $\frac{dy}{dx} = \frac{d}{dx}(x^{-2}\cos x - 4x^{-3})$
$= -2x^{-3}\cos x - x^{-2}\sin x + 12x^{-4}$
$= \frac{-2\cos x}{x^3} - \frac{\sin x}{x^2} + \frac{12}{x^4}$
$= \frac{12 - 2x\cos x - x^2\sin x}{x^4}$

23. $\frac{dy}{du} = \frac{d}{du}\left(\frac{\sin u}{\csc u} + \frac{\cos u}{\sec u}\right)$
$= \frac{d}{du}(\sin^2 u + \cos^2 u)$
$= \frac{d}{du}(1)$
$= 0$

24. $\frac{dy}{du} = \frac{d}{du}\left(\frac{\cot u}{\tan u} - \frac{\csc u}{\sin u}\right)$
$= \frac{d}{du}(\cot^2 u - \csc^2 u)$
$= \frac{d}{du}(-1)$
$= 0$

25. $\frac{dy}{dx} = \frac{d}{dx}[2x^{-2}(x^5 - x^3)]$
$= \frac{d}{dx}(2x^3 - 2x)$
$= 6x^2 - 2$

26. $\frac{dy}{dx} = \frac{d}{dx}[4x^2(x^{-1} + 3x^{-4})]$
$= \frac{d}{dx}(4x + 12x^{-2})$
$= 4 - 24x^{-3}$

27. $\frac{dy}{dt} = \frac{d}{dt}\left(\frac{t^2}{\pi^3} - \frac{\pi^2}{t^3}\right)$
$= \frac{d}{dt}\left(\frac{1}{\pi^3}t^2 - \pi^2 t^{-3}\right)$
$= \frac{2}{\pi^3}t + 3\pi^2 t^{-4}$
$= \frac{2t}{\pi^3} + \frac{3\pi^2}{t^4}$

28. $\frac{dy}{dt} = \frac{d}{dt}\left(\frac{t^3}{\pi^2} - \frac{\pi^3}{t^2}\right)$
$= \frac{d}{dt}\left(\frac{1}{\pi^2}t^3 - \pi^3 t^{-2}\right)$
$= \frac{3}{\pi^2}t^2 + 2\pi^3 t^{-3}$
$= \frac{3t^2}{\pi^2} + \frac{2\pi^3}{t^3}$

29. $\frac{dy}{dx} = \frac{d}{dx}(\sec x \tan x \cos x) = \frac{d}{dx}\tan x = \sec^2 x$

30. $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sin x \cot x}{\cos x}\right) = \frac{d}{dx}(1) = 0$

31. Since $y = \frac{\sin x}{x}$ is defined for all $x \neq 0$ and $\frac{dy}{dx} = \frac{x\cos x - \sin x}{x^2}$, the function is differentiable for all $x \neq 0$.

32. Since $y = \sin x - x\cos x$ is defined for all real x and
$\frac{dy}{dx} = \cos x - (x)(-\sin x) - (\cos x)(1) = x\sin x,$
the function is differentiable for all real x.

33. Since $y = \frac{3\cos x}{x-2}$ is defined for all $x \neq 2$ and $\frac{dy}{dx} = \frac{-3\sin x(x-2) - 3\cos x}{(x-2)^2}$, which is defined for all $x \neq 2$, the function is differentiable for all $x \neq 2$.

34. Since $y = (2x-7)^{-1}(x+5) = \dfrac{x+5}{2x-7}$ is defined for all $x \neq \dfrac{7}{2}$ and $\dfrac{dy}{dx} = \dfrac{(2x-7)(1)-(x+5)(2)}{(2x-7)^2} = -\dfrac{17}{(2x-7)^2}$, the function is differentiable for all $x \neq \dfrac{7}{2}$.

35. Slope $= \left.\dfrac{dy}{dx}\right|_{x=\pi} = \sec \pi \tan \pi = 0$

36. Slope $= \left.\dfrac{dy}{dx}\right|_{x=\pi} = \cos \pi \cos \pi - \sin \pi \sin \pi = 1$

37. Slope $= \left.\dfrac{dy}{dx}\right|_{x=\pi} = \dfrac{\pi(-\sin \pi) - 1 \cdot \cos \pi}{\pi^2} = \dfrac{1}{\pi^2}$

38. Slope $= \left.\dfrac{dy}{dx}\right|_{x=\pi}$

$$= \frac{1(\pi + \sin \pi) - (1 + \cos \pi)\pi}{\pi^2}$$
$$= \frac{\pi - 0}{\pi^2}$$
$$= \frac{1}{\pi}$$

39. $\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{\cos x}\right) = \dfrac{d}{dx}(\sec x) = \sec x \tan x$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(\sec x \tan x)$$
$$= (\sec x \tan x)\tan x + (\sec x)\sec^2 x$$
$$= \frac{\sin^2 x + 1}{\cos^3 x}$$

40. $\dfrac{dy}{dx} = \dfrac{d}{dx}(\csc x) = -\csc x \cot x$

$$\frac{d^2y}{dx^2}$$
$$= \frac{d}{dx}(-\csc x \cot x)$$
$$= -(-\csc x \cot x)\cot x + (-\csc x)(-\csc^2 x)$$
$$= \frac{\cos^2 x + 1}{\sin^3 x}$$

41. $\dfrac{dy}{dx} = \dfrac{d}{dx}(x \sin x)$

$$= 1 \cdot \sin x + x \cos x$$
$$= \sin x + x \cos x$$
$$\frac{d^2y}{dx^2} = \frac{d}{dx}(\sin x + x \cos x)$$
$$= \cos x + 1 \cdot \cos x + x(-\sin x)$$
$$= 2\cos x - x \sin x$$

42. $\dfrac{dy}{dx} = \dfrac{d}{dx}(x - x\cos x)$

$$= 1 - (1 \cdot \cos x + x(-\sin x))$$
$$= 1 - \cos x + x \sin x$$
$$\frac{d^2y}{dx^2} = \frac{d}{dx}(1 - \cos x + x \sin x)$$
$$= 0 - (-\sin x) + (1 \cdot \sin x + x \cdot \cos x)$$
$$= 2\sin x + x \cos x$$

43. $y' = 2x^3 - 3x - 1$,

$y'' = 6x^2 - 3$,

$y''' = 12x$,

$y^{(4)} = 12$, and the rest are all zero.

44. $y' = \dfrac{x^4}{24}$,

$y'' = \dfrac{x^3}{6}$,

$y''' = \dfrac{x^2}{2}$,

$y^{(4)} = x$,

$y^{(5)} = 1$, and the rest are all zero.

45. $\dfrac{dy}{dx} = \dfrac{d}{dx}(8x^{-2}) = -16x^{-3}$

At $x = 2$, $y = 8(2^{-2}) = 2$ and

$\dfrac{dy}{dx} = -16(2^{-3}) = -2.$

(a) Tangent: $y - 2 = -2(x - 2)$ or $y = -2x + 6$

(b) Normal: $y - 2 = \dfrac{1}{2}(x - 2)$ or $y = \dfrac{1}{2}x + 1$

46. $\dfrac{dy}{dx} = \dfrac{d}{dx}(4 + \cot x - 2\csc x)$

$$= -\csc^2 x + 2\csc x \cot x$$

At $x = \dfrac{\pi}{2}$,

$y = 4 + \cot\frac{\pi}{2} - 2\csc\frac{\pi}{2} = 4 + 0 - 2 = 2$ and

$$\frac{dy}{dx} = -\csc^2\frac{\pi}{2} + 2\csc\frac{\pi}{2}\cot\frac{\pi}{2}$$
$$= -1 + 2(1)(0)$$
$$= -1.$$

(a) Tangent:

$$y - 2 = -1\left(x - \frac{\pi}{2}\right) \text{ or } y = -x + \frac{\pi}{2} + 2$$

(b) Normal:

$$y - 2 = 1\left(x - \frac{\pi}{2}\right) \text{ or } y = x - \frac{\pi}{2} + 2$$

47. $\frac{dy}{dx} = \frac{d}{dx}(\sin x + \cos x) = \cos x - \sin x$

At $x = \frac{\pi}{4}$, $y = \sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \sqrt{2}$ and

$\frac{dy}{dx} = \cos\frac{\pi}{4} - \sin\frac{\pi}{4} = 0.$

(a) Tangent: Line is horizontal, so $y = \sqrt{2}$.

(b) Normal: Line is vertical, so $x = \frac{\pi}{4}$.

48. $\frac{dy}{dx} = \frac{d}{dx}\left(2x^2 + \frac{1}{x^4}\right) = 4x - 4x^{-5}$

At $x = 1$, $y = 2(1^2) + \frac{1}{1^4} = 3$ and

$\frac{dy}{dx} = 4(1) - 4(1)^{-5} = 0.$

(a) Tangent: Line is horizontal, so $y = 3$.

(b) Normal: Line is vertical, so $x = 1$.

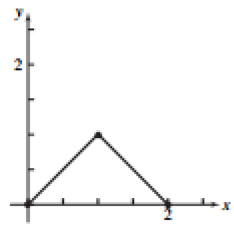
49. $\frac{dy}{dx} = 6x^2 = 6 \Rightarrow x = \pm 1$. The points are (1, 2) and (−1, −2).

50. $\frac{dy}{dx} = \frac{1}{6}(6x^2 - 6x) = 6 \Rightarrow x^2 - x = 6 \Rightarrow x = 3$ or $x = -2$. The points are $\left(3, \frac{9}{2}\right)$ and $\left(-2, -\frac{14}{3}\right)$.

51. $\frac{dy}{dx} = \frac{6(x+1) - 1 \cdot 6x}{(x+1)^2} = \frac{6}{(x+1)^2} = 6 \Rightarrow x = 0$ or $x = -2$. The points are (0, 0) and (−2, 12).

52. $\frac{dy}{dx} = 2\cos x = 6 \Rightarrow \cos x = 3$, which is impossible. There are no points at which the tangent line has slope 6, so "none."

53. (a)



(b) Yes, because both of the one-sided limits as $x \to 1$ are equal to $f(1) = 1$.

(c) No, because the left-hand derivative at $x = 1$ is + 1 and the right-hand derivative at $x = 1$ is − 1.

54. (a) For all m, since $y = \sin 2x$ and $y = mx$ are both continuous on their domains, and they link up at the origin, where $\lim_{x \to 0^-} \sin 2x = \lim_{x \to 0^+} mx = 0$, regardless of the value of m.

(b) For $m = 2$ only, since the left-hand derivative at 0 (which is 2 cos 0 = 2) must match the right-hand derivative at 0 (which is m).

55. Note that $\frac{dy}{dx} = \frac{4}{5}x^{-1/5} = \frac{4}{5\sqrt[5]{x}}$ is defined if and only if $x \neq 0$. The answers are

(a) For all $x \neq 0$

(b) At $x = 0$

(c) Nowhere

56. Note that $\frac{dy}{dx} = \frac{3}{5}x^{-2/5} = \frac{3}{5\sqrt[5]{x^2}}$ is defined if and only if $x \neq 0$. The answers are

(a) For all $x \neq 0$

(b) At $x = 0$

(c) Nowhere

57. Note that $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (2x-3) = -3$ and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (x-3) = -3$. Since these values agree with $f(0)$, the function is continuous at $x = 0$. On the other hand, $f'(x) = \begin{cases} 2, & -1 \le x < 0 \\ 1, & 0 < x \le 4 \end{cases}$, so the derivative is undefined at $x = 0$.

(a) $[-1, 0) \cup (0, 4]$

(b) At $x = 0$

(c) Nowhere in its domain

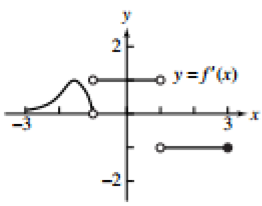
58. Note that the function is undefined at $x = 0$.

(a) $[-2, 0) \cup (0, 2]$

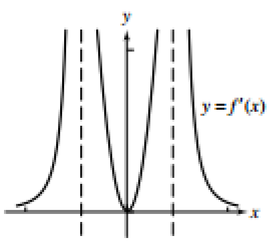
(b) Nowhere

(c) Nowhere in its domain

59.



60.

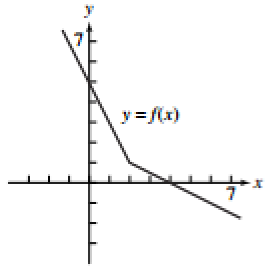


61. **(a)** iii

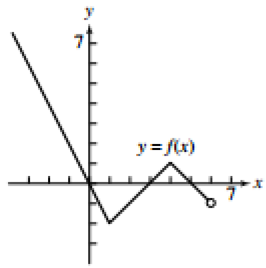
(b) i

(c) ii

62. The graph passes through (0, 5) and has slope –2 for $x < 2$ and slope –0.5 for $x > 2$.



63. The graph passes through (–1, 2) and has slope –2 for $x < 1$, slope 1for $1< x < 4$, and slope –1 for $4 < x < 6$.



64. **i.** If $f(x) = \frac{9}{28}x^{7/3} + 9$, then $f'(x) = \frac{3}{4}x^{4/3}$ and $f''(x) = x^{1/3}$, which matches the given equation.

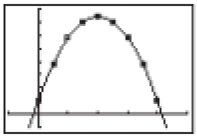
ii. If $f'(x) = \frac{9}{28}x^{7/3} - 2$, then $f''(x) = \frac{3}{4}x^{4/3}$, which contradicts the given equation.

iii. If $f'(x) = \frac{3}{4}x^{4/3} + 6$, then $f''(x) = x^{1/3}$, which matches the given equation.

iv. If $f(x) = \frac{3}{4}x^{4/3} - 4$, then $f'(x) = x^{1/3}$ and $f''(x) = \frac{1}{3}x^{-2/3}$, which contradicts the given equation.

Answer is D: i and iii only could be true. Note, however that i and iii could not simultaneously be true.

65. **(a)**

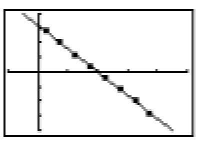


[-1, 5] by [-10, 80]

(b)

t interval	avg. vel.
[0, 0.5]	$\frac{38-10}{0.5-0}=56$
[0.5, 1]	$\frac{58-38}{1-0.5}=40$
[1, 1.5]	$\frac{70-58}{1.5-1}=24$
[1.5, 2]	$\frac{74-70}{2-1.5}=8$
[2, 2.5]	$\frac{70-74}{2.5-2}=-8$
[2.5, 3]	$\frac{58-70}{3-2.5}=-24$
[3, 3.5]	$\frac{38-58}{3.5-3}=-40$
[3.5, 4]	$\frac{10-38}{4-3.5}=-56$

(c)



[-1, 5] by [-80, 80]

(d) Average velocity is a good approximation to velocity.

66. $(x^n)'=nx^{n-1}$; $(x^n)''=n(n-1)x^{n-2}$; $(x^n)'''=n(n-1)(n-2)x^{n-3}$; ... and

$$\frac{d^n}{dx^n}(x^n)=n(n-1)(n-2)(n-3)\cdots 2\cdot 1x^0$$
$$=n!.$$

67. **(a)** $\left.\frac{d}{dx}(3f(x))\right|_{x=1}=3f'(1)=3\cdot 4=12$

(b) $\left.\frac{d}{dx}(xf(x))\right|_{x=1}=1\cdot f(x)+x\cdot f'(x)|_{x=1}$
$=f(1)+f'(1)$
$=-3+4$
$=1$

(c) $\left.\frac{d}{dx}(x^2f(x))\right|_{x=1}=2x\cdot f(x)+x^2\cdot f'(x)|_{x=1}$
$=2f(1)+f'(1)$
$=-6+4$
$=-2$

(d) $\left.\frac{d}{dx}\left(\frac{f(x)}{x}\right)\right|_{x=1}=\left.\frac{f'(x)\cdot x-1\cdot f(x)}{x^2}\right|_{x=1}$
$=\frac{f'(1)-f(1)}{1}$
$=4-(-3)$
$=7$

(e) $\left.\frac{d}{dx}\left(\frac{f(x)}{x^2+2}\right)\right|_{x=0}$
$=\left.\frac{f'(x)\cdot(x^2+2)-2x\cdot f(x)}{(x^2+2)^2}\right|_{x=0}$
$=\frac{f'(0)\cdot 2-0\cdot f(0)}{2^2}$
$=\frac{(-2)\cdot 2}{4}$
$=-1$

(f) $\left.\frac{d}{dx}(f(x)\cdot f(x))\right|_{x=0}$
$=f'(x)\cdot f(x)+f(x)\cdot f'(x)|_{x=0}$
$=2f'(0)f(0)$
$=2(-2)(9)$
$=-36$

68. **(a)** $\left.\frac{d}{dx}[3f(x)-g(x)]\right|_{x=-1}$
$=3f'(x)-g'(x)|_{x=-1}$
$=3(2)-1$
$=5$

(b) $\left.\frac{d}{dx}(f(x)g(x))\right|_{x=0}$
$=f'(x)\cdot g(x)+f(x)\cdot g'(x)|_{x=0}$
$=(-2)(-3)+(-1)(4)$
$=2$

(c) $\left.\frac{d}{dx}(f(x)g(x))\right|_{x=-1}$
$=f'(x)\cdot g(x)+f(x)\cdot g'(x)|_{x=-1}$
$=(2)(-1)+(0)(1)$
$=-2$

(d) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)\bigg|_{x=0}$
$= \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{(g(x))^2}\bigg|_{x=0}$
$= \frac{(-2)(-3)-(-1)(4)}{(-3)^2}$
$= \frac{10}{9}$

(e) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)\bigg|_{x=-1}$
$= \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{(g(x))^2}\bigg|_{x=-1}$
$= \frac{(2)(-1)-(0)(1)}{(-1)^2}$
$= -2$

(f) $\frac{d}{dx}\left(\frac{f(x)}{g(x)+2}\right)\bigg|_{x=0}$
$= \frac{f'(x)\cdot (g(x)+2) - f(x)\cdot g'(x)}{(g(x)+2)^2}\bigg|_{x=0}$
$= \frac{(-2)(-1)-(-1)(4)}{(-3+2)^2}$
$= 6$

69. Yes; the slope of $f+g$ at $x = 0$ is $(f+g)'(0) = f'(0) + g'(0)$. The sum of two positive numbers must also be positive.

70. No; it depends on the values of $f(0)$ and $g(0)$. For example, let $f(x) = x$ and $g(x) = x - 1$. Both lines have positive slope everywhere, but $(f\cdot g)(x) = x^2 - x$ has a negative slope at $x = 0$.

71. (a) $\frac{ds}{dt} = \frac{d}{dt}(64t - 16t^2) = 64 - 32t$

$\frac{d^2s}{dt^2} = \frac{d}{dt}(64 - 32t) = -32$

(b) The maximum height is reached when $\frac{ds}{dt} = 0$, which occurs at $t = 2$ sec.

(c) When $t = 0$, $\frac{ds}{dt} = 64$, so the velocity is 64 ft/sec.

(d) Since $\frac{ds}{dt} = \frac{d}{dt}(64t - 2.6t^2) = 64 - 5.2t$, the maximum height is reached at $t = \frac{64}{5.2} \approx 12.3$ sec. The maximum height is $s\left(\frac{64}{5.2}\right) \approx 393.8$ ft.

72. (a) Solving $160 = 490t^2$, it takes $\frac{4}{7}$ sec. The average velocity is $\frac{160}{\frac{4}{7}} = 280$ cm/sec.

(b) Since $v(t) = \frac{ds}{dt} = 980t$, the velocity is $(980)\left(\frac{4}{7}\right) = 560$ cm/sec. Since $a(t) = \frac{dv}{dt} = 980$, the acceleration is $980\ \text{cm}/\text{sec}^2$.

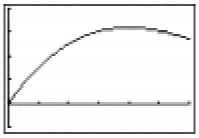
73. $\frac{dV}{dx} = \frac{d}{dx}\left[\pi\left(10 - \frac{x}{3}\right)x^2\right]$
$= \frac{d}{dx}\left[\pi\left(10x^2 - \frac{1}{3}x^3\right)\right]$
$= \pi(20x - x^2)$

74. (a) $r(x) = \left(3 - \frac{x}{40}\right)^2 x = 9x - \frac{3}{20}x^2 + \frac{1}{1600}x^3$

(b) The marginal revenue is
$r'(x) = 9 - \frac{3}{10}x + \frac{3}{1600}x^2$
$= \frac{3}{1600}(x^2 - 160x + 4800)$
$= \frac{3}{1600}(x-40)(x-120)$,
which is zero when $x = 40$ or $x = 120$. Since the bus holds only 60 people, we require $0 \le x \le 60$. The marginal revenue is 0 when there are 40 people, and the corresponding fare is

$p(40) = \left(3 - \frac{40}{40}\right)^2 = \$4.00.$

(c) One possible answer:
If the current ridership is less than 40, then the proposed plan may be good. If the current ridership is greater than or equal to 40, then the plan is not a good idea. Look at the graph of $y = r(x)$.



[0, 60] by [−50, 200]

75. (a) Since $x = \tan\theta$, we have

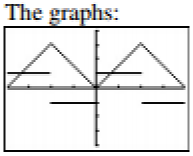
$\frac{dx}{dt} = (\sec^2\theta)\frac{d\theta}{dt} = -0.6\sec^2\theta$. At point A, we have

$\theta = 0$ and $\frac{dx}{dt} = -0.6\sec^2 0 = -0.6$ km/sec.

(b) 0.6

$\frac{\text{rad}}{\text{sec}} \cdot \frac{1 \text{ revolution}}{2\pi \text{ rad}} \cdot \frac{60\text{sec}}{1\text{min}} = \frac{18}{\pi}$ revolutions per minute or approximately 5.73 revolutions per minute.

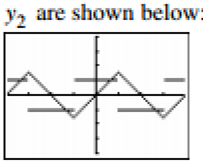
76. (a) The graphs:



[−6.3, 6.3] by [−4.1, 4.1]

It appears that the derivative of y_1 is y_2.

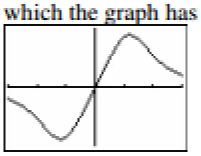
(b) Let $y_2 = \frac{|\cos(x)|}{\cos(x)}$. The graphs of y_1 and y_2 are shown below:



[−6.3, 6.3] by [−4.1, 4.1]

It again appears that the derivative of y_1 is y_2.

77. The graph of the function indicates that the range is confined between the two points at which the graph has horizontal tangents.



[−3, 3] by [−0.5, 0.5]

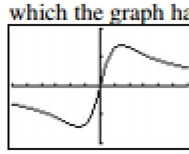
Setting

$$0 = \frac{dy}{dx} = \frac{3(x^4+6) - 4x^3(3x)}{(x^4+6)^2} = \frac{18-9x^4}{(x^4+6)^2},$$ we

get $x = \pm\sqrt[4]{2}$. Plugging these values back into the equation of the curve, we get

$\frac{3(\pm\sqrt[4]{2})}{2+6} = \pm\frac{3\sqrt[4]{2}}{8}$. Thus $a = \frac{3\sqrt[4]{2}}{8}$.

78. The graph of the function indicates that the range is confined between the two points at which the graph has horizontal tangents.



[−6, 6] by [−2, 2]

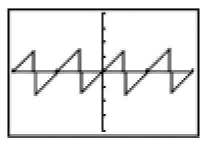
Setting

$$0 = \frac{dy}{dx} = \frac{4(x^2+2) - 2x(4x)}{(x^2+2)^2} = \frac{8-4x^2}{(x^2+2)^2},$$ we

get $x = \pm\sqrt{2}$. Plugging these values into the equation of the curve, we get $\frac{4(\pm\sqrt{2})}{2+2} = \pm\sqrt{2}$.

Thus $a = \sqrt{2}$.

79.



[−π, π] by [−4, 4]

(a) $x \neq k\frac{\pi}{4}$, where k is an odd integer

(b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(c) Where it's not defined, at $x = k\frac{\pi}{4}$, k an odd integer

(d) It has period $\frac{\pi}{2}$ and continues to repeat the pattern seen in this window.