

$$1. 2 \sin 100^\circ \cos 100^\circ = \sin 200^\circ$$

$$2. \frac{2 \tan 40^\circ}{1 - \tan^2 40^\circ} = \tan 2(40^\circ) = \tan 80^\circ$$

1. $\tan^2 x - \sin^2 x = \sin^2 x \tan^2 x$

$$\frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x \cos^2 x}{\cos^2 x} =$$

$$\frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x} =$$

$$\frac{\sin^2 x (\sin^2 x)}{\cos^2 x} =$$

$$\tan^2 x \sin^2 x \checkmark$$

5. $\cos 3x = 4 \cos^3 x - 3 \cos x$

\Rightarrow (Sum ID) $\cos(2x+x) =$

(Dbl. ID) $\cos 2x \cos x - \sin 2x \sin x =$

$$(\cos^2 x - \sin^2 x) \cos x - 2 \sin x \cos x \cdot \sin x$$

$$\cos^3 x - \sin^2 x \cos x - 2 \sin^2 x \cos x$$

(Pyth. ID.) $\cos^3 x - 3 \sin^2 x \cos x$

$$\cos^3 x - 3(1 - \cos^2 x) \cos x$$

$$\cos^3 x - 3 \cos x + 3 \cos^3 x$$

$$4 \cos^3 x - 3 \cos x \checkmark$$

(like terms)

$$9. \csc x - \cos x \cot x = \sin x$$

$$\Leftrightarrow \frac{1}{\sin x} - \cos x \cdot \frac{\cos x}{\sin x} =$$

$$\frac{1 - \cos^2 x}{\sin x} =$$

$$\frac{\sin^2 x}{\sin x} = \sin x \checkmark$$

$$11. \frac{1 + \tan \theta}{1 - \tan \theta} + \frac{1 + \cot \theta}{1 - \cot \theta} = 0$$

$\Leftrightarrow \mathbb{R}$
(common denominator)

$$\frac{(1 + \tan \theta)(1 - \cot \theta) + (1 + \cot \theta)(1 - \tan \theta)}{(1 - \tan \theta)(1 - \cot \theta)} =$$

$$\frac{1 - \cancel{\cot \theta} + \tan \theta - \tan \theta \cot \theta + 1 - \cancel{\tan \theta} + \cot \theta - \tan \theta \cot \theta}{1 - \cot \theta - \tan \theta + \tan \theta \cot \theta}$$

$$= \frac{0}{1 - \cot \theta - \tan \theta + 1} = 0 \checkmark$$

$\Leftrightarrow \mathbb{R}$

$$13. \cos^2\left(\frac{t}{2}\right) = \frac{1 + \sec t}{2 \sec t}$$

$$\left(\sqrt{\frac{1 + \cos t}{2}}\right)^2 =$$

$$\frac{1 + \cos t}{2} \cdot \frac{\sec t}{\sec t} = \frac{\sec t + 1}{2 \sec t} \checkmark$$

15. ~~15.~~
L \Rightarrow R

$$\frac{\cos x}{1 - \tan x} + \frac{\sin x}{1 - \cot x} = \cos x + \sin x$$

$$\frac{\cos x}{1 - \frac{\sin x}{\cos x}} + \frac{\sin x}{1 - \frac{\cos x}{\sin x}} =$$

$$\frac{\cos x}{\frac{\cos x - \sin x}{\cos x}} + \frac{\sin x}{\frac{\sin x - \cos x}{\sin x}} =$$

$$\frac{\cos^2 x}{\cos x - \sin x} + \frac{\sin^2 x}{\sin x - \cos x} =$$

$$\frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} =$$

$$\frac{(\cancel{\cos x - \sin x})(\cos x + \sin x)}{\cancel{\cos x - \sin x}} = \cos x + \sin x \checkmark$$

19. L \Rightarrow R
 $\tan(u + \frac{3\pi}{4}) = \frac{\tan u - 1}{1 + \tan u}$

$$\frac{\tan u + \tan \frac{3\pi}{4}}{1 - \tan u \tan \frac{3\pi}{4}} =$$

$$\frac{\tan u - 1}{1 + \tan u} \checkmark$$

$$[\tan \frac{3\pi}{4} = -1]$$

21. L \Rightarrow R
 $\tan \frac{1}{2} \beta = \csc \beta - \cot \beta$

$$\tan \frac{\beta}{2} =$$

$$\frac{1 - \cos \beta}{\sin \beta} =$$

$$\frac{1}{\sin \beta} - \frac{\cos \beta}{\sin \beta} =$$

$$\csc \beta - \cot \beta \checkmark$$

$$39. 2\cos x = 1$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$41. \sin^2 x - 2\sin x - 3 = 0$$

$$(\sin x + 1)(\sin x - 3) = 0$$

$$\sin x = -1, 3$$

$$x = \frac{3\pi}{2}$$

$$42. \cos 2t = \cos t$$

$$2\cos^2 t - 1 = \cos t$$

$$2\cos^2 t - \cos t - 1 = 0$$

$$(2\cos t + 1)(\cos t - 1) = 0$$

$$\cos t = -\frac{1}{2}, 1$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}, 0$$

$$44. \cos 2x + 5\cos x = 2$$

$$2\cos^2 x - 1 + 5\cos x = 2$$

$$2\cos^2 x + 5\cos x - 3 = 0$$

$$(2\cos x - 1)(\cos x + 3) = 0$$

$$\cos x = \frac{1}{2}, -3$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$