Pre-Calculus Review Ch. 3.1-3.5 Name DiMarco

For 1-2, compute the exact value of the function for the given x-value. (NO CALC)

1. $f(x) = -3\cdot 4^x,\ x = -\frac{1}{2}$

$f(-\frac{1}{2}) = -3(4)^{-\frac{1}{2}} = -3(\frac{1}{2}) = -\frac{3}{2}$

2. $f(x) = 6\cdot 3^x,\ x = \frac{3}{2}$

$f(\frac{3}{2}) = 6(3)^{3/2} = 6(\sqrt{27}) = 6(3\sqrt{3}) = 18\sqrt{3}$

Write the equation of an exponential function that goes through the given points. (CALC OK)

3. (0, 3) and (5, 90)

(0,a) (x,y)

$y = a\cdot b^x$

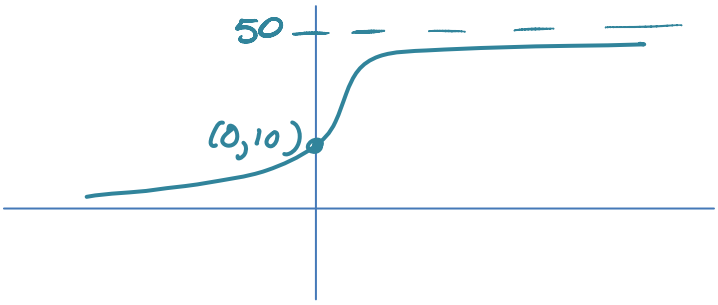
$90 = 3\cdot b^5$

$30 = b^5$

$b = 30^{(\frac{1}{5})} = 1.97$

$f(x) = 3(1.97)^x$

4. Find the y-intercept, horizontal asymptotes, and ~~line of symmetry~~ of the logistic function and sketch a graph (NO CALC) $f(x) = \frac{50}{1+4\cdot 0.3^x}$



H.A. $y = 0$, $y = 50$

y-int: $f(0) = \frac{50}{1+4(.3)} = \frac{50}{5} = 10$

(0,10)

5. Find the equation of a logistic function that has an initial value of 18, a limit to growth of 30, and passes through the point (3, 25). (CALC OK)

(0,18) $c = 30$ (x,y)

① $18 = \frac{30}{1+a\cdot b^0}$

$18 = \frac{30}{1+a}$

$18+18a = 30$

$18a = 12$ $a = \frac{2}{3}$

② $25 = \frac{30}{1+\frac{2}{3}b^3}$

$25 + \frac{50}{3}b^3 = 30$

$\frac{3}{50}\cdot\frac{50}{3}b^3 = 5\cdot\frac{3}{50}$

$b^3 = \frac{15}{50}$ $b = .669$

$f(x) = \frac{30}{1+\frac{2}{3}(.669)^x}$

Numbers 6 and 7 are CALC OK.

6. The population of Elmhurst is 45,000 in the year 2000 and is decreasing by 1.8% each year.

a) Write an equation that models the population as a function of time t in years.

$P(t) = 45{,}000(1-.018)^t = 45000(.982)^t$

b) What will be the population in the year 2015?

$t = 15$ $P(15) = 45{,}000(.982)^{15} = 34{,}268$

(or use Graph, Value X=15)

c) Predict when the population will be 20,000.

$y_2 = 20{,}000$

$X = 44.6$ years 2045

7. The population P of elk after t years in Blackberry State Park is modeled by the logistic function $P(t) = \frac{1200}{1+99e^{-0.4t}}$

a) What was the initial population of elk?

y-int $P(0) = \frac{1200}{1+99e^{-0.4(0)}} = \frac{1200}{100} =$ **12 elk**

b) When will there be 1000 elk in the park?

$y_2 = 1000$

$x =$ **15.5 years**

c) What is the maximum number of elk that the park can sustain?

1200 elk (c-value)

8. Evaluate each log expression without using a calculator. (NO CALC)

a) $\log_2 32$ $= 5$

b) $\log_3 \frac{1}{81}$ -4

c) $\log \sqrt[3]{10}$ $\frac{1}{3}$

d) $\ln \frac{1}{\sqrt{e^3}}$ $-\frac{3}{2}$

9. Assuming x and y are positive use properties of logs to expand the logarithm.

a) $\log \frac{3+x}{4+y}$ $\log(3+x) - \log(4+y)$

b) $\log_2 x^{-3}$ $-3\log_2 x$

c) $\ln 1000x^3$ $\ln 1000 + \ln x^3$ $= \ln 1000 + 3\ln x$

d) $\log_6 \sqrt{\frac{x}{y^4}}$ $\frac{1}{6}\log\frac{x}{y^4} = \frac{1}{6}(\log x - \log y^4)$ $= \frac{1}{6}(\log x - 4\log y)$

10. Assuming x, y, and z are all positive, use properties of logs to write each expression as a single log.

a) $3\log(x+2) - 4\log(x-4)$ $\log(x+2)^3 - \log(x-4)^4$ $\log\frac{(x+2)^3}{(x-4)^4}$

b) $3\ln(x-1) + \ln(x+2)$ $\ln(x-1)^3 + \ln(x+2)$ $\ln(x-1)^3(x+2)$

Solve each of the following (NO CALC!):

11. $2(10^{-3x}) = 200$

$10^{-3x} = 100$

$10^{-3x} = 10^2$

$-3x = 2$ $x = -\frac{2}{3}$

12. $16\left(\frac{1}{2}\right)^{\frac{x}{4}} = 2$

$\left(\frac{1}{2}\right)^{x/4} = \frac{1}{8}$

$\left(\frac{1}{2}\right)^{\frac{x}{4}} = \left(\frac{1}{2}\right)^3$ $\frac{x}{4} = 3$ $x = 12$

13. $\log_3 x = -2$

$3^{-2} = x$

$x = \frac{1}{9}$

Solve each of the following (CALC OK):

14. $1.06^x = 4.1$

$\log_{1.06} 4.1 = x$

$x = \frac{\log 4.1}{\log 1.06} \approx 24.22$

15. $50e^{.035x} = 200$

$e^{.035x} = 4$

$\ln 4 = .035x$

$x = \frac{\ln 4}{.035} \approx 39.61$

16. $3 + 2e^{-x} = 6$

$2e^{-x} = 3$

$e^{-x} = \frac{3}{2}$

$\ln \frac{3}{2} = -x$

$x = -\ln \frac{3}{2} \approx -.41$

17. $\log_4(x-5) = -1$

$4^{-1} = x - 5$

$x = 4^{-1} + 5 = \frac{1}{4} + 5$

$= 5\frac{1}{4} \approx 5.25$

18. $3\log(x-3) + 4 = 5$

$3\log(x-3) = 1$

$\log(x-3) = \frac{1}{3}$

$10^{\frac{1}{3}} = x - 3$

$x = 10^{\frac{1}{3}} + 3 \approx 5.15$

19. $\ln(x-3) + \ln(x+4) = 3\ln 2$

$\ln(x-3)(x+4) = \ln 8$

$x^2 + x - 12 = 8$

$x^2 + x - 20 = 0$

$(x-4)(x+5) = 0$

$x = 4, -5$