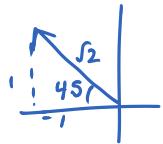


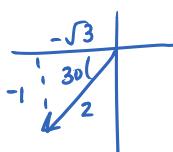
PART I: 4.1-4.3, 5.5, 5.6 (Right Angle Trig, Unit Circle, Law of Sines & Cosines)

(#1-6) Without using a calculator, find the exact values of each:

1. $\cos 135^\circ = \boxed{-\frac{\sqrt{2}}{2}}$

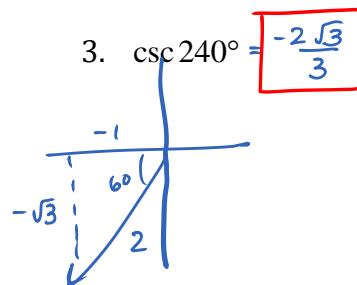


2. $\cot \frac{7\pi}{6} = \boxed{\sqrt{3}}$

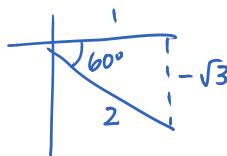


4. $\sin \pi = \boxed{0}$

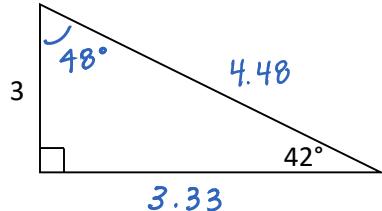
5. $\sec 90^\circ = \boxed{\text{undefined}}$



6. $\tan \frac{5\pi}{3} = \boxed{-\sqrt{3}}$



7. Solve for the missing angles and sides of the triangle. **



$\sin 42^\circ = \frac{3}{h}$

$h = 4.48$

$\cos 42^\circ = \frac{x}{4.48}$

$x = 3.33$

8. Convert 37 degrees to radians. **

$37^\circ \times \frac{\pi}{180^\circ} = \boxed{\frac{37\pi}{180}}$

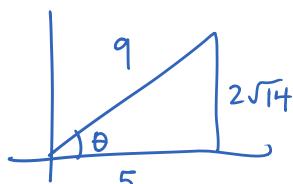
9. Convert 2 radians to degrees. **

$2 \times \frac{180^\circ}{\pi} = \boxed{\frac{360^\circ}{\pi} \text{ or } 114.59^\circ}$

10. Assume the angle θ is an acute angle. Find the other five trig. functions if:

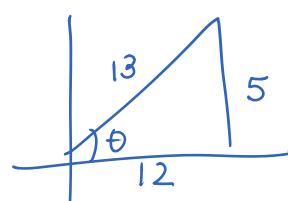
a. $\cos \theta = \frac{5}{9}$

b. $\csc \theta = \frac{13}{5}$



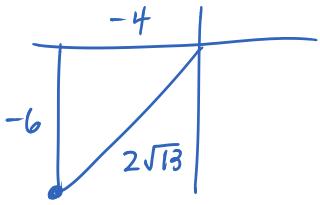
$$\begin{aligned} 5^2 + y^2 &= 9^2 \\ y^2 &= 56 \\ y &= 2\sqrt{14} \end{aligned}$$

$$\begin{aligned} \sin \theta &= \frac{2\sqrt{14}}{9} & \csc \theta &= \frac{9}{2\sqrt{14}} \\ \sec \theta &= \frac{9}{5} & \cot \theta &= \frac{5}{2\sqrt{14}} \\ \tan \theta &= \frac{2\sqrt{14}}{5} \end{aligned}$$



$$\begin{aligned} \sin \theta &= \frac{5}{13} & \cos \theta &= \frac{12}{13} & \sec \theta &= \frac{13}{12} \\ \tan \theta &= \frac{5}{12} & \cot \theta &= \frac{12}{5} \end{aligned}$$

11. Evaluate the six trig. functions if point P (-4, -6) is on the terminal side of an angle θ .



$$4^2 + 6^2 = h^2 \\ 52 = h^2 \\ 2\sqrt{13} = h$$

$$\sin \theta = \frac{-6}{2\sqrt{13}} = \frac{-3}{\sqrt{13}}$$

$$\csc \theta = -\frac{\sqrt{13}}{3}$$

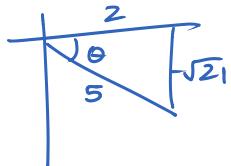
$$\cos \theta = \frac{-4}{2\sqrt{13}} = \frac{-2}{\sqrt{13}}$$

$$\sec \theta = -\frac{\sqrt{13}}{2}$$

$$\tan \theta = \frac{-6}{-4} = \frac{3}{2}$$

$$\cot \theta = \frac{2}{3}$$

12. Find $\sin \theta$ and $\tan \theta$ if $\cos \theta = \frac{2}{5}$ and $\cot \theta < 0$.



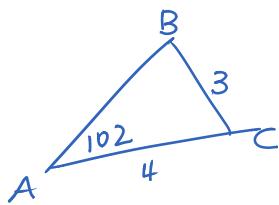
$\text{quad 1 or 4} =$

$\text{quad 2 or 4} =$

$$\sin \theta = -\frac{\sqrt{21}}{5} \quad \tan \theta = -\frac{\sqrt{21}}{2}$$

13. Determine if the triangle has 0, 1, or 2 possible triangles. **

a. $a = 3, b = 4, A = 102^\circ$



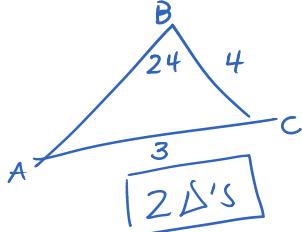
$$\frac{\sin 102}{3} = \frac{\sin B}{4}$$

$$\sin B = 4 \frac{\sin 102}{3}$$

$$\sin B = 1.30$$

NO Δ's

b. $a = 4, b = 3, B = 24^\circ$



$$\frac{\sin 24}{3} = \frac{\sin A}{4}$$

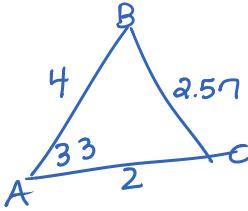
$$\sin A = 4 \frac{\sin 24}{3}$$

$$A = 32.84$$

$$\text{Supp. of } A = 147.16$$

14. Solve the triangle and find the area of the triangle. **

a. $A = 33^\circ, b = 2, c = 4$



$$a^2 = 4^2 + 2^2 - 2(4)(2)\cos 33^\circ$$

$$a = 2.57$$

$$\frac{\sin 33}{2.57} = \frac{\sin B}{2}$$

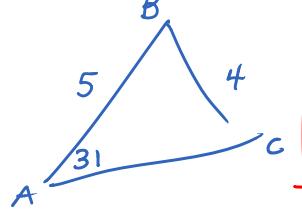
$$B = 25.13^\circ$$

$$\text{Area} = \frac{1}{2}(4)(2)\sin 33^\circ$$

$$= 2.18 \text{ units}^2$$

$$C = 121.87^\circ$$

b. $A = 31^\circ, a = 4, c = 5$



$$\frac{\sin 31}{4} = \frac{\sin C}{5}$$

$$C = 40.08^\circ$$

$$B = 108.92^\circ$$

$$\frac{\sin 108.92}{b} = \frac{\sin 31}{4}$$

$$b = 7.35$$

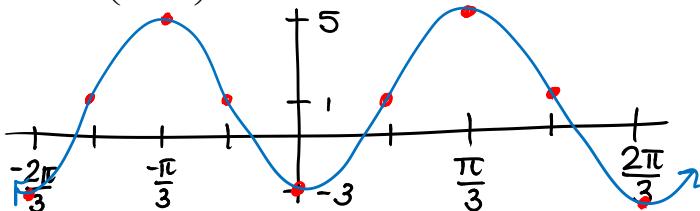
$$\text{Area} = \frac{1}{2}(5)(4)\sin 31^\circ$$

$$\text{Area} = 9.46 \text{ units}^2$$

PART II: 4.4, 4.5, 4.7 (Trig Graphs, Inverse Trig, Solving Trig Equations)

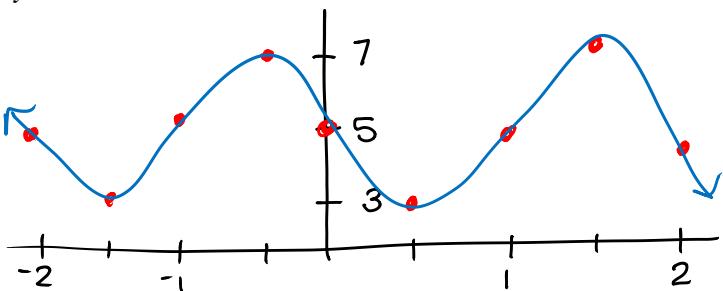
(#15-18) Sketch two periods of the graph of the trig function. Make sure to include your scale and critical values on each axis.

15. $y = 4 \cos 3\left(x - \frac{\pi}{3}\right) + 1$



$$P = \frac{2\pi}{3}$$

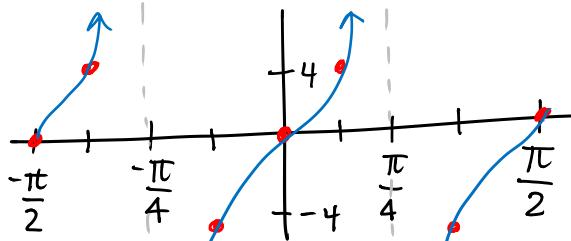
16. $y = -2 \sin \pi x + 5$



$$P = \frac{2\pi}{\pi} = 2$$

#2
 $\angle C = 139.92^\circ$
 $\angle B = 9.08^\circ$
 $b = 1.23$

17. $y = 4 \tan 2x$

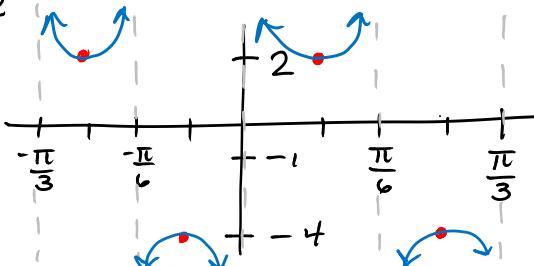


$$P = \frac{\pi}{2}$$

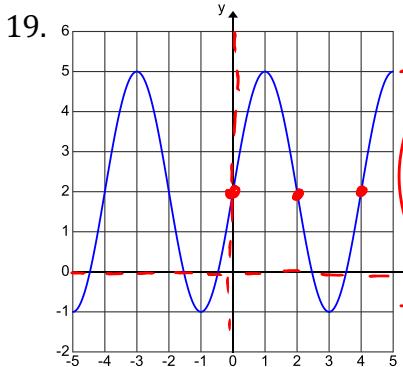
18. $y = 3 \csc 6x - 1$



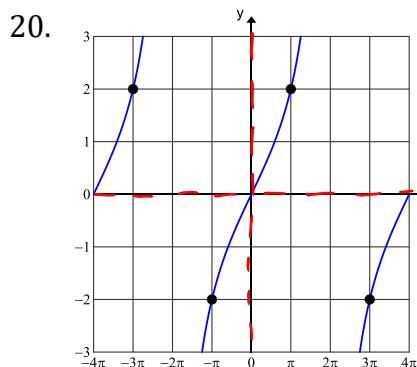
$$P = \frac{2\pi}{6} = \frac{\pi}{3}$$



(#19-20) Write the equation of the each graph shown below. (4 points each)



$$y = 3 \sin \frac{\pi}{2} x + 2$$



$$y = 2 \tan \frac{1}{4} x$$

(#21-25) Solve the trig equation over the interval $[0, 2\pi]$.

21. $\cos x = -\frac{\sqrt{2}}{2}$

$$x = 45^\circ \Rightarrow \text{quad 2 \& 3}$$

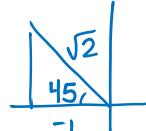
$$x = \frac{3\pi}{4}, \frac{5\pi}{4}$$

22. $\cot x = -\sqrt{3}$
quad 2 & 4



$$x = \frac{5\pi}{6}, \frac{11\pi}{6}$$

23. $\sec x = -\sqrt{2}; [-\pi, \pi]$
quad 2 \& 3



$$x = \frac{3\pi}{4}, \frac{5\pi}{4}$$

24. $\sin x = 0.73^{**}$
quad 1 \& 2

$$x = .82, 2.32$$

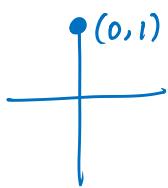
25. $\sec x = -1.92^{**}$
quad 2 \& 3

$$\cos^{-1}(-1.92)$$

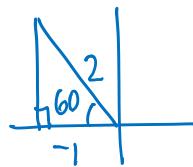
$$x = 2.12, 4.16$$

(#26-30) evaluate the inverse trig function. Reminder: inverse trig functions have restricted domains!

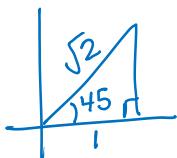
26. $\sin^{-1}(1) = \frac{\pi}{2}$



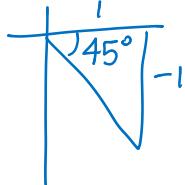
27. $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$



28. $\sec^{-1}(\sqrt{2}) = 45^\circ$

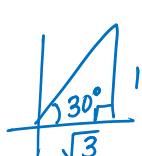


29. $\tan^{-1}(-1) = -\frac{\pi}{4}$



30. $\cos\left(\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)\right)$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$



31. Write the equation of a sine graph that has an amplitude of 4, a period of length 3π , a phase shift of $\frac{\pi}{4}$ to the left, and a vertical shift down 2. $\downarrow 2$

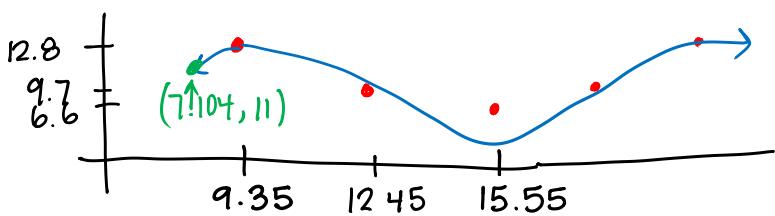
$$a = 4$$

$$3\pi = \frac{2\pi}{b}$$

$$b = \frac{2}{3}$$

$$y = 4 \sin \frac{2}{3}(x + \frac{\pi}{4}) - 2$$

32. At Hilton Head Island on June 10, high tide measured 12.8 feet on a pier at 9:21 am. The next low tide measured 6.6 feet at 3:33 pm. Write a sinusoidal equation modeling the behavior of the tide. What is the first time on June 10 that the tide measures 11 feet? ** $P = 12.4 = \frac{2\pi}{b} = \frac{\pi}{6.2}$

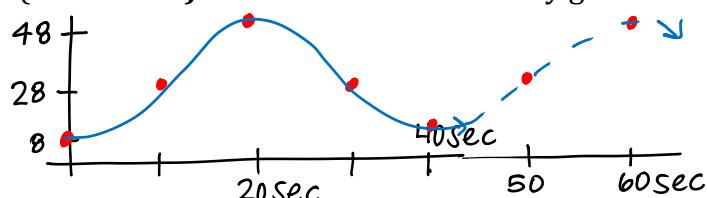


$$y = 3.1 \cos \frac{\pi}{6.2}(x - 9.35) + 9.7$$

approximately 7:06am

33. The Ferris wheel at a local amusement park has a diameter of 40 feet and reaches a maximum height of 48 feet above the ground. One ride is three revolutions, which takes 2 minutes to complete. **

- a. Draw a sketch of the graph and create an equation to model the height of a rider in terms of time (in seconds) on the Ferris wheel if they get on the ride at the bottom.



$$\frac{2}{3} = \frac{2\pi}{b}$$

$$b = 3\pi$$

$$\frac{2 \text{ min}}{3 \text{ rev}} = \frac{2 \text{ min}}{3} / \text{rev} \approx 40 \text{ sec}$$

$$y = -20 \cos 3\pi x + 28$$

- b. How high is the ride after 20 seconds? After 1 minute?

48 feet

- c. At what time(s) during the full ride does the rider reach a height of 25 feet?

$$.15, .52, .82, 1.18, 1.48, 1.85$$

$$9 \text{ sec}, 31.2 \text{ sec}, 49.2 \text{ sec}, 70.8 \text{ sec}, 88.8 \text{ sec}, 111 \text{ sec}$$

PART III: Chapter 5 (Trig Identities)

(#34-37) Simplify using trig identities:

34. $\cos^3 x + \cos x \sin^2 x$

$$\frac{\cos x (\cos^2 x + \sin^2 x)}{\cos x} = 1$$

36. $\frac{1}{\sin^2 x} + \frac{\sec^2 x}{\tan^2 x}$

$$\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} = \frac{2}{\sin^2 x} = 2 \csc^2 x$$

(#38-41) Find all solutions in the interval $[0, 2\pi]$.

38. $\sqrt{2} \cot x \sin x - \cot x = 0$

$$\cot x (\sqrt{2} \sin x - 1) = 0$$

$$\begin{aligned}\cot x &= 0 & \sin x &= 1/\sqrt{2} \\ x &= \frac{\pi}{2}, \frac{3\pi}{2} & x &= \frac{\pi}{4}, \frac{3\pi}{4}\end{aligned}$$

40. $\sin 2x - 2 \sin x = 0$

$$\begin{aligned}2 \sin x \cos x - 2 \sin x &= 0 \\ 2 \sin x (\cos x - 1) &= 0 \\ 2 \sin x &= 0 & \cos x &= 1 \\ x &= 0, \pi & x &= 0\end{aligned}$$

$$35. \frac{\cos^2 u + \cot^2 u + \sin^2 u}{\csc u}$$

$$\frac{1 + \cot^2 u}{\frac{1}{\sin u}} = \frac{1}{\sin^2 u} \cdot \sin u$$

$$\frac{1}{\sin u} \Rightarrow \boxed{\csc u}$$

$$37. \frac{1 + \cot \theta}{1 + \tan \theta} = \frac{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}}$$

$$= \frac{\sin x + \cos x}{\sin x} \cdot \frac{\cos x}{\cos x + \sin x}$$

$$\cot x$$

39. $3 \cos t = 2 \sin^2 t$

$$\begin{aligned}3 \cos t - 2(1 - \cos^2 t) &= 0 \\ 2 \cos^2 t + 3 \cos t - 2 &= 0\end{aligned}$$

$$(2 \cos t - 1)(\cos t + 2) = 0$$

$$\begin{aligned}\cos t &= 1/2 & \cos t &= -2 \\ t &= \frac{\pi}{3}, \frac{5\pi}{3}\end{aligned}$$

41. $\cos 2x = \sin x$

$$\begin{aligned}\cos 2x - \sin x &= 0 \\ (1 - 2 \sin^2 x) - \sin x &= 0 \\ -2 \sin^2 x - \sin x + 1 &= 0 \\ -(2 \sin^2 x + \sin x - 1) &= 0 \\ -(2 \sin x - 1)(\sin x + 1) &= 0 \\ \sin x &= 1/2 & \sin x &= -1\end{aligned}$$

(#42-45) Prove the following:

$$42. \underbrace{\cos x + \sec x}_{\cos x} = \frac{2 - \sin^2 x}{\cos x}$$

$$\begin{aligned} & \frac{\cos x}{\cos x} \frac{\cos x}{\cos x} + \frac{1}{\cos x} \\ & \frac{\cancel{\cos^2 x} + 1}{\cos x} = \frac{(1 - \sin^2 x) + 1}{\cos x} \\ & = \frac{2 - \sin^2 x}{\cos x} \quad \checkmark \end{aligned}$$

$$44. \underbrace{\frac{1}{1 - \cos t}}_{\sim} = \frac{1 + \cos t}{\sin^2 t}$$

$$\begin{aligned} &= \frac{(1 + \cos t) 1}{(1 + \cos t)(1 - \cos t)} \\ &= \frac{1 + \cos t}{1 - \cos^2 t} = \frac{1 + \cos t}{\sin^2 t} \quad \checkmark \end{aligned}$$

(#46-49) Evaluate the following without a calculator, using either the Sum/Difference or Half-Angle identities.

$$46. \sin 105^\circ$$

$$\begin{aligned} & \sin(60 + 45) \\ & \sin 60 \cos 45 + \cos 60 \sin 45 \\ & \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \end{aligned}$$

$$\frac{\sqrt{6} + \sqrt{2}}{4}$$

$$48. \tan\left(\frac{\pi}{12}\right)$$

$$\tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$\frac{\tan \frac{\pi}{3} - \tan \frac{\pi}{4}}{1 + \tan \frac{\pi}{3} \tan \frac{\pi}{4}} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \quad x = \frac{3\pi}{2}$$

$$43. \underbrace{1 + \tan^2 x}_{\sim} = \frac{1}{1 - \sin^2 x}$$

$$\begin{aligned} &= \sec^2 x \\ &= \frac{1}{\cos^2 x} \\ &= \frac{1}{1 - \sin^2 x} \quad \checkmark \end{aligned}$$

$$45. \cos\left(x - \frac{3\pi}{2}\right) = -\sin x$$

$$\begin{aligned} &= \cos x \cos \frac{3\pi}{2} + \sin x \sin \frac{3\pi}{2} \\ &= \cos x (0) + \sin x (-1) \\ &= -\sin x \quad \checkmark \end{aligned}$$

$$47. \cos(-75^\circ)$$

$$\cos(60 - 135)$$

$$\begin{aligned} & \cos(60) \cos(135) + \sin(60) \sin(135) \\ & \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \end{aligned}$$

$$\boxed{\frac{-\sqrt{2} + \sqrt{6}}{4}}$$

$$49. \sin\left(\frac{5\pi}{8}\right)$$

$$\sin\left(\frac{\frac{5\pi}{4}}{2}\right) = +\sqrt{\frac{1 - \cos \frac{5\pi}{4}}{2}}$$

$$= \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2 + \sqrt{2}}{2}}$$

