Precalculus

Q1 Cumulative Review – Chapter P, 1 and 2

****Denotes "No Calc" problems***

Name: Key

CHAPTER P

Write the following in interval notation. **

1. x is a positive number

   $(0, \infty)$

2. $0 < x < 3$

   $(0, 3)$

3. $x \le -6$ or $x > -5$

   $(-\infty, -6] \cup (-5, \infty)$

Solve the following equations for x. **

4. $x(3x+2) = 20$

   $3x^2 + 2x - 20 = 0$

   $x = \dfrac{-2 \pm \sqrt{4 - 4(3)(-20)}}{6}$

   $x = \dfrac{-2 \pm \sqrt{244}}{6} = \dfrac{-2 \pm 2\sqrt{61}}{6} = \boxed{\dfrac{-1 \pm \sqrt{61}}{3}}$

5. $4x^2 = 64$

   $x^2 = 16$

   $\boxed{x = \pm 4}$

6. $6x^2 + 7x - 3 = 0$

   $(3x - 1)(2x + 3) = 0$

   $\boxed{x = 1/3, -3/2}$

7. $2x^2 - 5x + 1 = 0$

   $x = \dfrac{5 \pm \sqrt{25 - 4(2)(1)}}{4}$

   $\boxed{x = \dfrac{5 \pm \sqrt{17}}{4}}$

Solve the inequality and sketch the graph on a number line. **

8. $|2x+1| > 3$

   $2x + 1 > 3$ $\quad$ $2x + 1 < -3$

   $2x > 2$ $\quad$ $2x < -4$

   $x > 1$ $\quad$ $x < -2$

   -2 $\quad$ 1

   $\boxed{(-\infty, -2) \cup (1, \infty)}$

9. $9x^2 \le 81x$

   $9x^2 - 81x \le 0$

   $9x(x - 9) \le 0$

   -1 $\quad$ 0 $\quad$ 9 $\quad$ 10

   $+$ $\quad$ $-$ $\quad$ $+$

   $\boxed{[0, 9]}$

10. $5x^3 + 14x^2 - 3x \ge 0$

    $x(5x^2 + 14x - 3) \ge 0$

    $x(5x - 1)(x + 3) \ge 0$

    -4 $\quad$ -1 $\quad$ $1/10$

    $-$ $\quad$ -3 $\quad$ $+$ $\quad$ 0 $\quad$ $-$ $\quad$ $1/5$ $\quad$ $+$

    $\boxed{[-3, 0] \cup [1/5, \infty)}$

Solve the equations and inequalities using a graphing calculator.

11. $|3x+7| = x^2 + 2x + 3$

    $x \approx -1.56, 2.56$

12. $\dfrac{1}{2}x + 2 = 3x^3 - x$

    $x \approx 1.06$

13. $3 = |-x^2 + 2x + 5|$

    $x \approx -2, -.73, 2.73, 4$

14. $x^3 - 2x^2 + x - 4 < -1$

    $(-\infty, 2.17)$

15. $x + 1 \ge -4$

    $[5, \infty)$

16. $x^2 + 2x - 3 > 5$

    $(-\infty, 4) \cup (2, \infty)$

State whether the equation is a function. **

Vertical Line test!

17. $y = \sqrt{x-4}$

Function

18. $y = 50x + 413$

Function

19. $y = |2x|$

Function

20. $x^2 + y^2 = 16$

Relation

CHAPTER 1

Use the equation $f(x) = x^4 - 3x^3 + x - 1$ to find its properties listed below.

21. Absolute maximum: n/a

22. Absolute minimum: $y = -7.32$ @ $x = 2.20$

23. Local maximum(s): $y = -.76$ @ $x = .36$

24. Local minimum(s): $y = -7.32$ @ $x = 2.20$

25. Increasing intervals: $[-.31, .36] \cup [2.20, \infty)$

26. Decreasing intervals: $(-\infty, -.31] \cup [.36, 2.20]$

27. Constant intervals: n/a

28. Even/odd/neither? neither

Find the domain of the following functions. **

29. $f(x) = x + \sqrt{x-4}$

$x - 4 \geq 0$

$x \geq 4$

$\boxed{[4, \infty)}$

30. $f(x) = x^2 - 3x + 4$

$\mathbb{R}$

31. $f(x) = \dfrac{1}{x\sqrt{4-x^2}}$

$x \neq 0$ $\quad$ $4 - x^2 > 0$

$(2+x)(2-x) > 0$

$-3 \quad -2 \quad 0 \quad 2 \quad 3$

$\ominus \quad \oplus \quad \oplus \quad \ominus$

$\boxed{(-2, 0) \cup (0, 2)}$

Use the equation $f(x) = -3x^2 - 2$. **

32. Is the function bounded/bounded above/bounded below/neither? *Circle one.*

33. Is the function even/odd/neither? *Circle one.*

34. Describe the transformations taking place (in order). $\boxed{y = x^2}$

vert.

① reflect over x-axis

② stretch b a f o 3

③ ↓ 2

OR ② → ① → ③

Use the functions to evaluate the following: $f(x)=\sqrt{x^2-9}$, $g(x)=2x+3$ **

35. $f+g$ $= \sqrt{x^2-9} + 2x+3$

36. $f-g$ $\sqrt{x^2-9} - 2x-3$

37. $f(g(x))$ $= \sqrt{(2x+3)^2-9}$

$= \sqrt{4x^2+12x+9-9}$

$= \boxed{\sqrt{4x^2+12x}}$

38. $g(f(x))$ $= \boxed{2(\sqrt{x^2-9})+3}$

39. Confirm the following two functions are inverses of one another: $f(x)=\frac{1}{2}x^3+4$, $g(x)=\sqrt[3]{2x-8}$ **

$f(g(x)) = \frac{1}{2}(\sqrt[3]{2x-8})^3+4 = \frac{1}{2}(2x-8)+4 = x-4+4 = x$ ✓

$g(f(x)) = \sqrt[3]{2(\frac{1}{2}x^3+4)-8} = \sqrt[3]{x^3+8-8} = \sqrt[3]{x^3} = x$ ✓

40. Find the inverse of $f(x)=\sqrt{x-1}+4$ and state the domain of $f^{-1}(x)$.**

range: $[4,\infty)$

$x = \sqrt{y-1}+4$

$x-4 = \sqrt{y-1}$

$x^2-8x+16+1 = y$

$\boxed{f^{-1}(x) = x^2-8x+17 \quad d: [4,\infty)}$

Use the function $h(x)=-3|x-2|+7$

41. List the parent functions and the transformations (in order) taking place. **

Vertical

① fup over x-axis

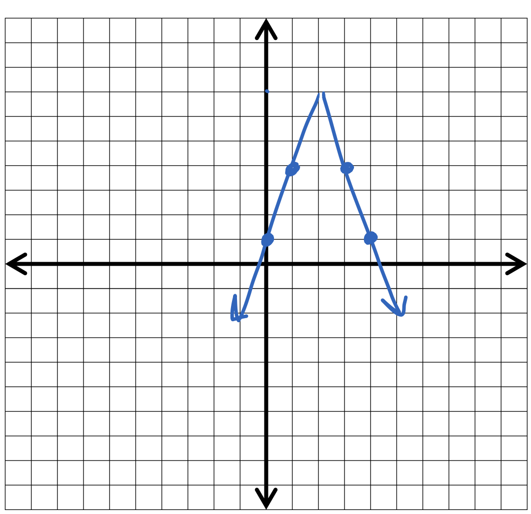
② stretch bafo 3

③ ↑7

Horizontal

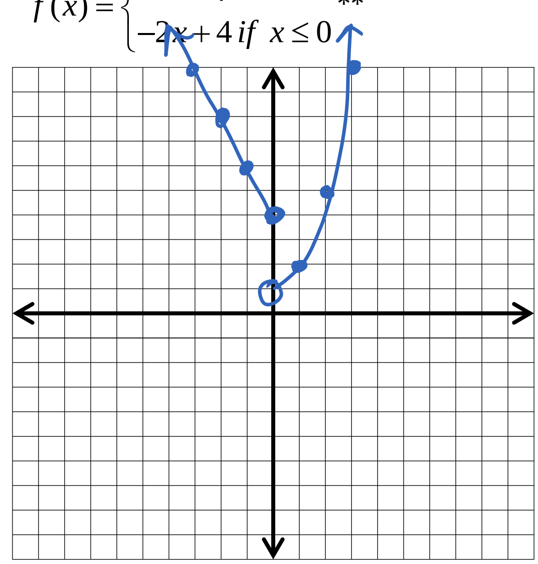
① →2

42. Graph $h(x)$. Plot at least 4 accurate points. **



43. Graph the following piecewise function:

$$f(x)=\begin{cases} x^2+1 \, if \;\; x>0 \\ -2x+4 \, if \;\; x\le 0 \end{cases}$$ **



CHAPTER 2

44. Find the vertex and axis of symmetry: $y = 3x^2 + 12x - 1$**

$x = \frac{-12}{2(3)} = \boxed{x = -2 \quad \text{Vertex}: (-2, -13)}$

$3(-2)^2 + 12(-2) - 1$

$3(4) - 24 - 1$

45. Convert #44 to vertex form by completing the square.**

$y = 3(x^2 + 4x + \underline{4}) - 1 - \underline{12}$

$\boxed{y = 3(x+2)^2 - 13}$

46. Write an equation of the line passing through (-2 , 7) and (2 , -1).**

$m = \frac{-1-7}{2+2} = \frac{-8}{4} = -2$

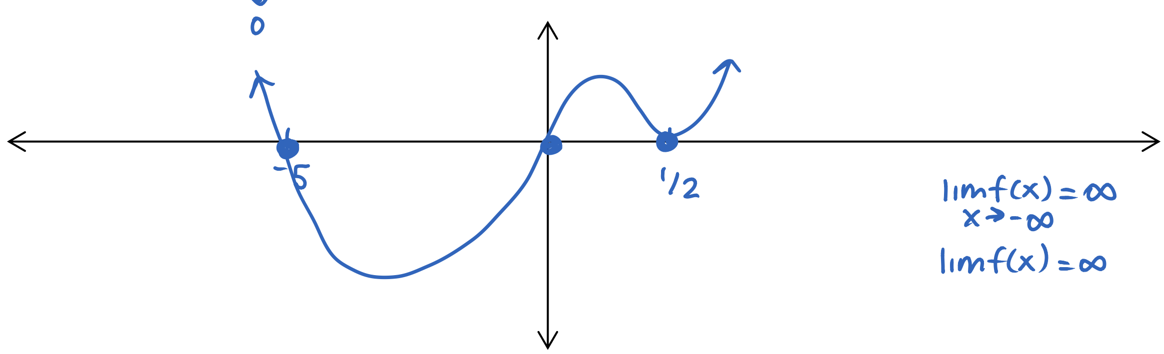
$\boxed{\begin{array}{c} y - 7 = -2(x+2) \\ \text{OR} \\ y + 1 = -2(x-2) \\ \text{OR} \\ y = -2x + 3 \end{array}}$

47. Sketch a graph of the following polynomial: $f(x) = x(2x-1)^2(x+5)^3$. Include zeros (with multiplicity), end behavior and the y-intercept **

0(1) 1/2(2) -5(3)

LT: $4x^6$

0



$\lim\limits_{x \to -\infty} f(x) = \infty$

$\lim f(x) = \infty$

$x \to -\infty$

48. Factor completely. Then write a linear factorization of the function (factored form):

$g(x) = x^5 - 3x^4 - 5x^3 + 5x^2 - 6x + 8 \Rightarrow \boxed{g(x) = (x+2)(x-1)(x-4)(x+i)(x-i)}$

zeros from calc: -2, 1, 4

$$\begin{array}{r|rrrrr|r} -2 & 1 & -3 & -5 & 5 & -6 & 8 \\ & \downarrow & -2 & 10 & -10 & 10 & -8 \\ \hline & 1 & -5 & 5 & -5 & 4 & 0 \end{array}$$

$$\begin{array}{r|rrrr|r} 1 & 1 & -5 & 5 & -5 & 4 \\ & \downarrow & 1 & -4 & 1 & -4 \\ \hline & 1 & -4 & 1 & -4 & 0 \end{array}$$

$$\begin{array}{r|rrr|r} 4 & 1 & -4 & 1 & -4 \\ & \downarrow & 4 & 0 & 4 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$x^2 + 1 = 0$

$x^2 = -1$

$x = \pm i$

49. Write a polynomial of minimum degree in factored form, then in standard form, that has zeros of 4 and 1 + 2i. **

zeros: $4,\ 1+2i,\ 1-2i$

factored form: $f(x) = (x-4)(x-(1+2i))(x-(1-2i))$

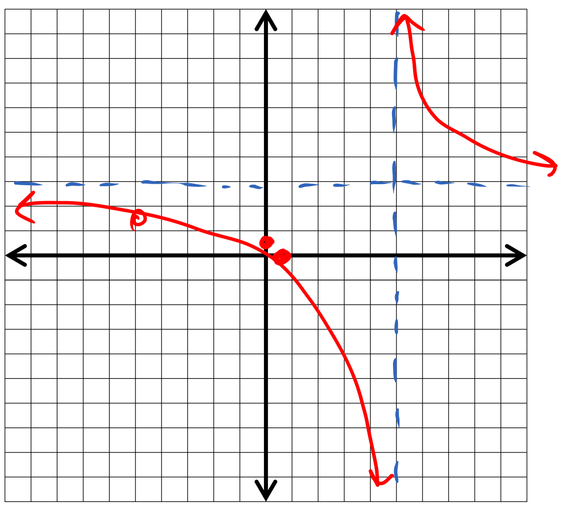
	x	-1	$-2i$
x	x^2	$-x$	$\cancel{-2xi}$
-1	$-x$	1	$\cancel{2i}$
$+2i$	$\cancel{2xi}$	$\cancel{-2i}$	$-4i^2 = 4$

$= (x-4)(x^2-2x+5)$

	x	-4
x^2	x^3	$-4x^2$
$-2x$	$-2x^2$	$8x$
$+5$	$5x$	-20

standard form: $f(x) = x^3 - 6x^2 + 13x - 20$

50. Graph the function. Include any asymptotes (vertical, horizontal, slant), removable discontinuities, x- and y-intercepts, and end behavior. $g(x) = \dfrac{3x^2+13x-10}{x^2-25}$ ** $\Rightarrow g(x) = \dfrac{(3x-2)\cancel{(x+5)}}{(x-5)\cancel{(x+5)}}$



V.A: $x=5$

H.A.: $y=3$

S.A.: n/a

R.D.: $(-5, 17/10)$

x-int: $(2/3, 0)$

y-int: $(0, 2/5)$

$\lim_{x\to 5^-} g(x) = -\infty$

$\lim_{x\to 5^+} g(x) = \infty$

$\lim_{x\to -\infty} g(x) = 3$

$\lim_{x\to \infty} g(x) = 3$

51. Solve for x. Check for extraneous solutions. $\dfrac{x}{x-2} + \dfrac{1}{x-4} = \dfrac{2}{\underset{(x-4)(x-2)}{x^2-6x+8}}$ **

$x(x-4) + 1(x-2) = 2$

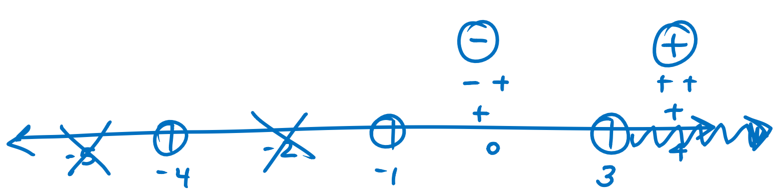
$x^2 - 4x + x - 2 - 2 = 0$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = \cancel{4}\ \boxed{-1}$

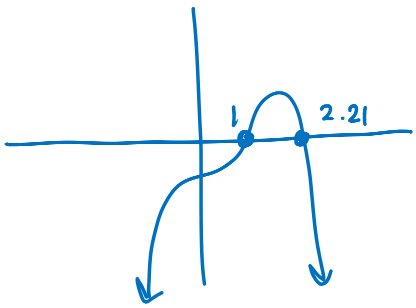
52. Solve the inequality. Create a sign chart! $\dfrac{(x-3)|x+4|}{\sqrt{x+1}} > 0$ **



$\boxed{(3, \infty)}$

53. Solve the inequality using your graphing calculator: $-x^4+3x^3-2x^2+x-1<0$

$\boxed{(-\infty,\ 1) \cup (2.21, \infty)}$

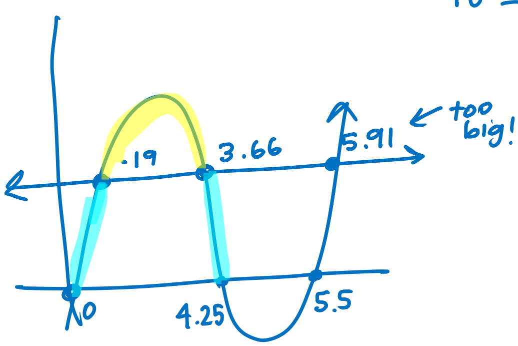


54. **Using a pair of scissors, you cut congruent squares off of the four corners of an 8.5" by 11" piece of card stock. Once the squares are cut off, you fold up the sides to form an open box (a box without a top).**

a. If you want the box to have a volume of <u>at least</u> 16 cubic inches, what size squares could have been cut from the cardstock?

$16 \le x(8.5-2x)(10-2x)$

$\boxed{[.19, 3.66]}$



b. If you want the box to have a volume <u>no more than</u> 16 cubic inches, what size squares could have been cut from the cardstock?

$16 \ge x(8.5-2x)(10-2x)$

$\boxed{(0, .19] \cup [3.66, 4.25)}$