

** Marks calculator OK

Chapter 4.1-4.3, 5.5, 5.6 (Right triangle and unit circle trig., Law of Sines and Cosines)

1. Find the exact values of each:

a) $\sin 300^\circ = \boxed{-\frac{\sqrt{3}}{2}}$

b) $\cot \frac{7\pi}{6} = \boxed{\sqrt{3}}$

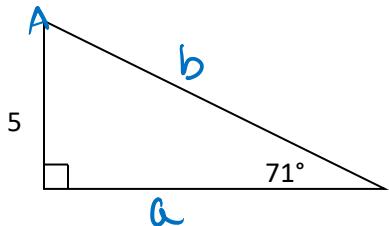
c) $\sec 225^\circ = \boxed{-\sqrt{2}}$

d) $\tan \pi = \boxed{0}$

e) $\csc \frac{\pi}{2} = \boxed{1}$

f) $\tan \frac{5\pi}{3} = \boxed{-\sqrt{3}}$

**2. Solve for missing angles and sides of the triangle:



$A = 19^\circ$

$\sin 71^\circ = \frac{5}{b}$

$b = \frac{5}{\sin 71^\circ} \approx \boxed{5.288}$

$\tan 71^\circ = \frac{5}{a}$

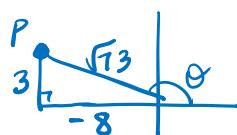
$a = \frac{5}{\tan 71^\circ} \approx \boxed{1.722}$

3. Convert 108 degrees to radians.

$108^\circ \left(\frac{\pi}{180^\circ} \right) = \boxed{\frac{3\pi}{5}}$

4. Convert 10 radians to degrees.

$10 \left(\frac{180^\circ}{\pi} \right) = \boxed{\frac{1800^\circ}{\pi}}$

5. Evaluate the six trig. functions if point P (-8, 3) is on the terminal side of an angle θ .

$\sqrt{8^2 + 3^2} = \sqrt{73}$

$\sin \theta = \frac{3}{\sqrt{73}}$

$\cos \theta = \frac{-8}{\sqrt{73}}$

$\tan \theta = -\frac{3}{8}$

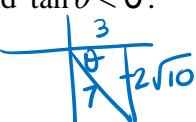
$\cot \theta = -\frac{8}{3}$

$\csc \theta = \frac{\sqrt{73}}{3}$

$\sec \theta = -\frac{\sqrt{73}}{8}$

6. Find $\sin \theta$ and $\cot \theta$ if $\cos \theta = \frac{3}{7}$ and $\tan \theta < 0$.

$\cos + \quad \tan -$
Q1(4) Q2(4)

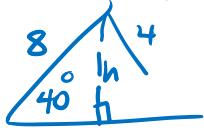


$\sin \theta = -\frac{2\sqrt{10}}{7}$

$\cot \theta = -\frac{3}{2\sqrt{10}}$

**7. Determine if the triangle has 0, 1, or 2 possible triangles:

a) $m\angle C = 40^\circ, a = 8, c = 4$



$h = 8 \sin 40^\circ = 5.14 > 4$
0 triangles

b) $m\angle H = 50^\circ, a = 12, h = 10$



$h = 12 \sin 50^\circ = 9.19 < 10$
2 triangles

**8. Solve the triangle and find the area of the triangle:

a) $A = 88^\circ, b = 6, c = 7$ SAS - Cosines

$a^2 = 6^2 + 7^2 - 2(6)(7)\cos 88^\circ$
 $a = 9.059$

$\frac{\sin 88^\circ}{9.059} = \frac{\sin B}{6}$
 $\angle B = 41.45^\circ$
 $\angle C = 50.55^\circ$

$A = \sqrt{s(s-a)(s-b)(s-c)}$
 $s = \frac{a+b+c}{2} = 11.029$
 $A = 20.987$

b) $t = 9, a = 4, c = 6$ SSS - Cosines

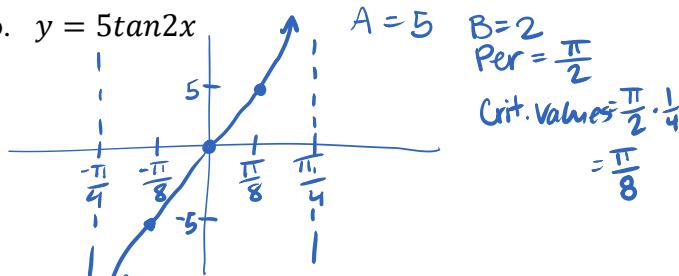
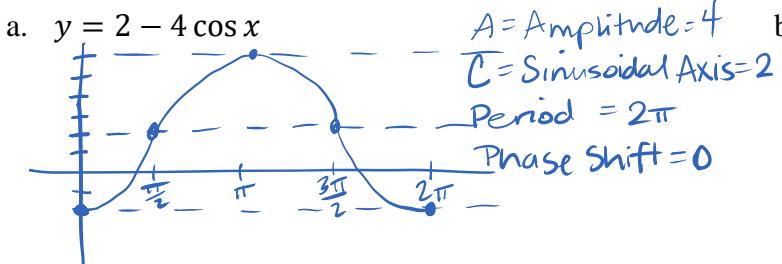
$q^2 = 4^2 + 6^2 - 2(4)(6)\cos T$
 $81 = 52 - 48 \cos T$
 $T = \cos^{-1}\left(-\frac{29}{48}\right)$
 $T = 127.17^\circ$

$\frac{\sin 127.17^\circ}{9} = \frac{\sin A}{4}$
 $\angle A = 20.74^\circ$
 $\angle C = 32.09^\circ$

$A = \frac{1}{2}(4)(9)\sin C$
 $A = 9.56$

Sections 4.4, 4.5, 4.7 (Trig Graphs, Inverse Trig)

9. Sketch the following graphs and describe their characteristics:



10. For the equation $y = -5 \sin \frac{\pi}{3}(x + \frac{\pi}{4}) - 9$, find the amplitude, vertical shift, period and phase shift.

$$\text{Amplitude} = 5$$

$$\text{Vertical shift} = -9$$

$$B = \frac{\pi}{3} \quad \text{Period} = \frac{2\pi}{\pi/3} = 2\pi \cdot \frac{3}{\pi} = 6$$

$$\text{Phase Shift} = -\frac{\pi}{4}$$

11. **A Ferris wheel 32 feet in diameter makes one revolution every 40 seconds. If the bottom of the wheel is 6 feet above the ground, write the equation to model the path of the rider on the wheel. Then, use the equation to find the first three times a rider is 25 feet above the ground (assume he gets on at the bottom).**

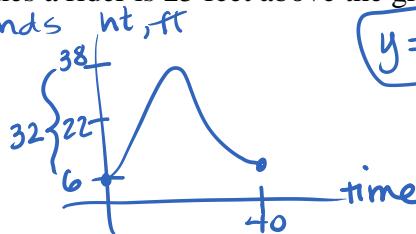
$$\text{Period} = 40 \text{ seconds}$$

$$B = \frac{2\pi}{40} = \frac{\pi}{20}$$

$$\text{radius} = A = 16$$

$$C = 22$$

$$D = 0$$



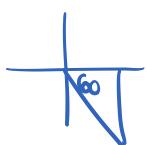
$$y = -16 \cos(\frac{\pi}{20}x) + 22$$

$$y_2 = 25$$

$$t = 11.2, 28.8, 51.2 \text{ seconds}$$

12. Find the following (give answers in radians):

$$\sin^{-1}(-\frac{\sqrt{3}}{2})$$



$$-\frac{\pi}{3}$$

$$\cos^{-1}(-\frac{\sqrt{2}}{2})$$



$$\frac{3\pi}{4}$$

$$\tan^{-1}(-\sqrt{3})$$



$$\cot^{-1}(-1)$$



$$\frac{3\pi}{4}$$

Chapter 5 (Trig Identities, Solving Trig Equations)

13. Simplify the following:

$$\text{a. } \frac{\cos^2 x + \sin^2 x}{\tan x} = \frac{1}{\tan x} = \boxed{\cot x}$$

$$\text{b. } \frac{1 + \tan^2 x}{\csc^2 x} = \frac{\sec^2 x}{\csc^2 x} = \frac{\sin^2 x}{\cos^2 x} = \boxed{\tan^2 x}$$

14. Solve the following over $[0, 2\pi]$:

$$\text{a. } 1 - \sqrt{2} \cos x = 0$$

$$-\sqrt{2} \cos x = -1 \quad \cos x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{7\pi}{4}$$

$$\text{c. } \sin 2x - \cos x = 0$$

$$2 \sin x \cos x - \cos x = 0$$

$$\cos x (2 \sin x - 1) = 0$$

$$\cos x = 0 \quad 2 \sin x - 1 = 0$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\text{b. } 2 \sin^2 x + \sin x - 1 = 0$$

$$(2 \sin x - 1)(\sin x + 1) = 0$$

$$\sin x = \frac{1}{2} \quad \sin x = -1$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

$$\text{d. } 3 \tan^2 x - 1 = 0$$

$$\tan^2 x = \frac{1}{3}$$

$$\tan x = \pm \frac{1}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

15. Prove the following:

a. $\csc x - \sin x = \cot x \cos x$

$$\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} =$$

$$\frac{\cos^2 x}{\sin x} =$$

$$\sqrt{\cot x \cdot \cos x} = \cot x \cos x$$

c. $\frac{(1-\cos x) \sin x}{(1-\cos x)(1+\cos x)} = \csc x - \cot x$

$$\frac{(1-\cos x) \sin x}{1-\cos^2 x} =$$

$$\frac{(1-\cos x) \sin x}{\sin^2 x} =$$

$$\frac{1-\cos x}{\sin x} =$$

$$\frac{1}{\sin x} - \frac{\cos x}{\sin x} =$$

b. $\cot x(\tan x \sin x + \cos x) = \csc x$

$$\cot x \cdot \tan x \sin x + \cot x \cdot \cos x =$$

$$1 \cdot \sin x + \frac{\cos x}{\sin x} \cdot \cos x =$$

$$\sin x + \frac{\cos^2 x}{\sin x} =$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x} =$$

d. $\frac{\cos 3x}{\cos(2x+x)} = \frac{1}{\csc x} = \csc x$

$$\cos 2x \cos x - \sin 2x \sin x =$$

$$(\cos^2 x - \sin^2 x) \cos x - 2 \sin x \cos x \sin x =$$

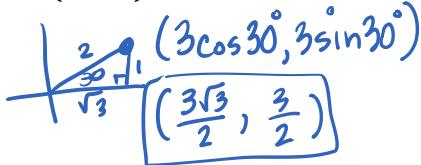
$$\cos^3 x - \sin^2 x \cos x - 2 \sin^2 x \cos x =$$

$$\cos^3 x - 3 \sin^2 x \cos x \checkmark$$

Chapter 6 (Vectors, Parametric Equations, Polars, Complex Plane)

16. Convert the following coordinates from polar to rectangular or from rectangular to polar:

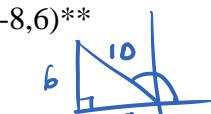
a. $(3, 30^\circ)$



b. $(-8, 265^\circ) \text{ **}$

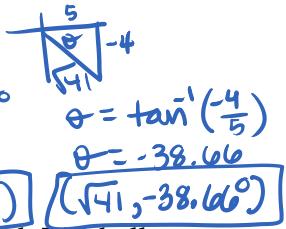
$$(-8 \cos 265^\circ, -8 \sin 265^\circ) \\ = (-0.70, 7.97)$$

c. $(-8, 6) \text{ **}$



$$x = \tan^{-1}\left(\frac{6}{8}\right) = 36.87^\circ \\ \theta = 180 - x = 143.13^\circ \\ (10, 143.13^\circ)$$

d. $(5, -4) \text{ **}$



17. **At the driving range, a golf ball is struck from an elevated tee box 8 feet above the ground. The ball was struck with an initial velocity of 180 ft./sec. at an angle of 38° to the horizontal.

a. Write the parametric equations for the flight of the ball.

$$x = 180 \cos(38^\circ) t$$

$$y = -16t^2 + 180 \sin(38^\circ) t + 8$$

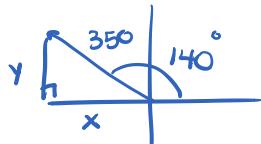
b. Find when and where the ball will be at its highest point.

$$199.89 \text{ ft high} \\ \text{at } t = 3.4 \text{ sec}$$

c. Determine when and where the ball will strike the ground.

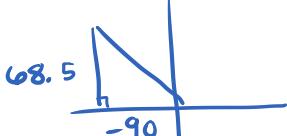
$$t = 7 \text{ sec} \\ x = 992.5 \text{ ft}$$

18. **Find the component form of a vector with a magnitude of 350 and a direction angle of 140° .



$$\langle 350 \cos 140^\circ, 350 \sin 140^\circ \rangle \\ \langle -268.12, 224.98 \rangle$$

19. **Find the direction angle and magnitude of a vector with a component form $\langle -90, 68.5 \rangle$.

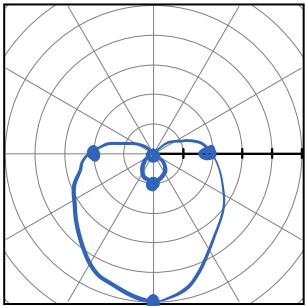


$$|v| = \sqrt{90^2 + 68.5^2} = 113.10$$

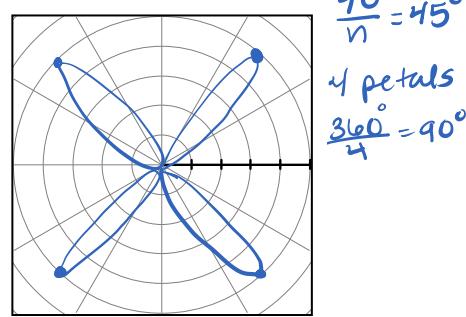
$$x = \tan^{-1}\left(\frac{68.5}{90}\right) = 37.28^\circ \\ \theta = 180 - x = 142.72^\circ$$

20. Graph the following polar equations.

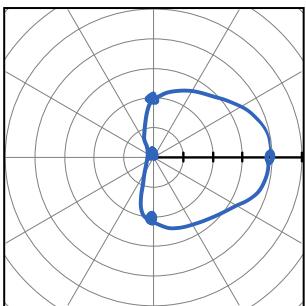
a. $r = 2 - 3\sin\theta$ **Limaçon**



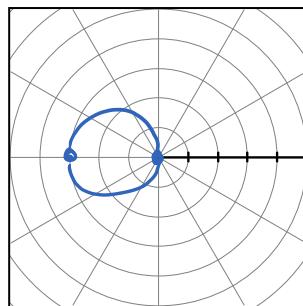
b. $r = 5\sin(2\theta)$ **Rose**



a. $r = 2 + 2\cos\theta$ **Cardioid**



b. $r = -3\cos\theta$ **Circle**



21. Write the equation to match the polar described below:

- a. Write the equation of a limaçon that has x-intercepts at 4 and -4, and y-intercepts at 5 and -3. $a=4$ $a+b=5$ $b=1$



$$r = 4 + 1 \sin\theta$$

- b. Write the equation of a circle that lies on the negative y-axis, with y-intercepts of 0 and -10.



$$r = -10 \sin\theta$$

22. Eliminate the Parameter: $x = 2t^2 + 3$; $y = t - 1$

$$\begin{aligned} x-3 &= 2t^2 \\ \frac{x-3}{2} &= t^2 \\ t &= \pm\sqrt{\frac{x-3}{2}} \end{aligned}$$

$$y = \pm\sqrt{\frac{x-3}{2}} - 1$$

23. Use De Moivre's Theorem to simplify $(2\sqrt{3} - 2i)^4$. Express your answer in standard form.

$$\begin{aligned} &\text{Diagram showing } z = 2\sqrt{3} - 2i \text{ in the second quadrant.} \\ r &= \sqrt{2^2 + (2\sqrt{3})^2} \\ r &= 4 \\ \theta &= \frac{11\pi}{6} \end{aligned}$$

$$\begin{aligned} &(4 \operatorname{cis} \frac{11\pi}{6})^4 = \left[4 \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right) \right]^4 \\ &= 4^4 \operatorname{cis} \frac{44\pi}{6} = 4^4 \operatorname{cis} \frac{22\pi}{3} \\ &= 256 \operatorname{cis} \frac{4\pi}{3} \quad \text{(same terminal side)} \\ &= 256 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right) \\ &= 256 \left(-\frac{1}{2} - i \frac{\sqrt{3}}{2} \right) = -128 - 128i\sqrt{3} \end{aligned}$$

Conics

24. Convert the following parametric equation of a conic section into general form:

center $(2, 4)$ ellipse $a=6$ $b=3$

$$\frac{(x-2)^2}{9} + \frac{(y-4)^2}{36} = 1$$

$$x = 2 + 3 \cos t$$

$$y = 4 + 6 \sin t$$

25. Determine the shape of the conic section and identify all important information before graphing. (If necessary, convert the following equations into general form by completing the square.)

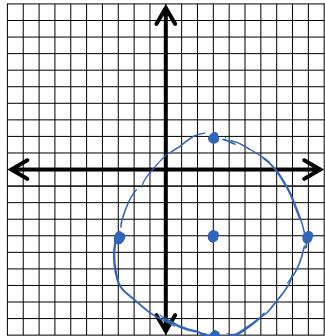
a. $x^2 - 6x + y^2 + 8y - 11 = 0$ Circle

$$x^2 - 6x + 9 + y^2 + 8y + 16 = 11 + 9 + 16$$

$$(x-3)^2 + (y+4)^2 = 36$$

center $(3, -4)$

$$r = 6$$



c. $\frac{(y-3)^2}{9} - \frac{(x+2)^2}{25} = 1$

Hyperbola

center $(-2, 3)$

opens up/down

$$a=3$$

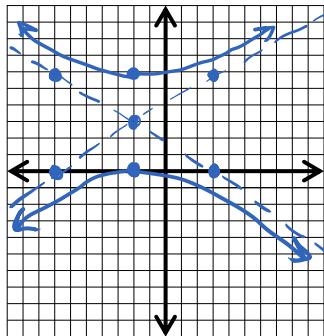
$$b=5$$

$$\text{slope of asymptotes} = \pm \frac{3}{5}$$

$$\text{foci } f^2 = 25 + 9 = 34$$

$$f = \sqrt{34}$$

$$\text{coords} = (-2, 3 \pm \sqrt{34})$$



b. $y^2 + 4y - 4x - 8 = 0$ Parabola

$$y^2 + 4y + 4 = 4x + 8 + 4$$

$$(y+2)^2 = 4x + 12$$

$$(y+2)^2 = 4(x+3)$$

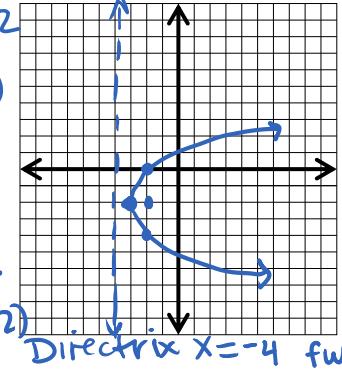
vertex $(-3, -2)$

$$4p = 4$$

$$p = 1$$

opens Right

focus $(-2, -2)$



d. $x^2 + 2x + 2y^2 - 8y = 7$ Ellipse

$$x^2 + 2x + 1 + 2(y^2 - 4y + 4) = 7 + 1 + 8$$

$$\frac{(x+1)^2}{16} + \frac{2(y-2)^2}{16} = 1$$

$$\frac{(x+1)^2}{16} + \frac{(y-2)^2}{8} = 1$$

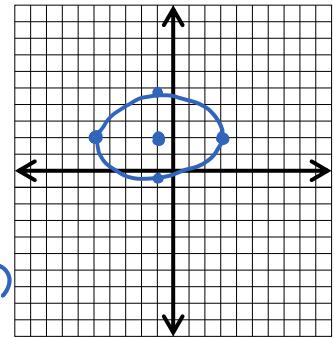
center $(-1, 2)$

$a=4$ (horizontal)

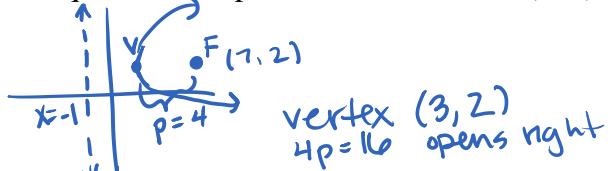
$$b=\sqrt{8}$$

$$\text{foci : } f^2 = 16 - 8 = 8 \quad \text{coords } (-1 \pm 2\sqrt{2}, 2)$$

$$f = \sqrt{8} = 2\sqrt{2}$$

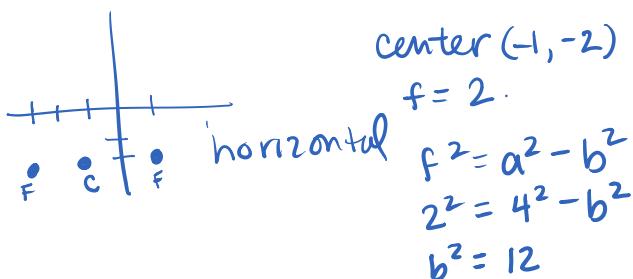


26. Find the equation for a parabola with focus at $(7, 2)$ and directrix $x = -1$.



$$(y-2)^2 = 16(x-3)$$

27. Find the equation of an ellipse with foci at $(-3, -2)$ and $(1, -2)$ and major axis length = 8.



$$\begin{aligned} 2a &= 8 \\ a &= 4 \end{aligned}$$

$$\frac{(x+1)^2}{16} + \frac{(y+2)^2}{12} = 1$$