6. $9x^2 - 16y^2 = (3x - 4y)(3x + 4y)$

7. $\dfrac{z^2 - 25}{z^2 - 5z} = \dfrac{(z-5)(z+5)}{z(z-5)} = \dfrac{z+5}{z}$

8. $\dfrac{x^2 + 2x - 35}{x^2 - 10x + 25} = \dfrac{(x+7)(x-5)}{(x-5)(x-5)} = \dfrac{x+7}{x-5}$

9. $\dfrac{x}{x-1} + \dfrac{x+1}{3x-4}$

$= \dfrac{x(3x-4)}{(x-1)(3x-4)} + \dfrac{(x+1)(x-1)}{(x-1)(3x-4)}$

$= \dfrac{4x^2 - 4x - 1}{(x-1)(3x-4)}$

10. $\dfrac{2x-1}{(x-2)(x+1)} + \dfrac{x-3}{(x-2)(x-1)}$

$= \dfrac{(2x-1)(x-1) + (x-3)(x+1)}{(x-2)(x+1)(x-1)}$

$= \dfrac{(2x^2 - 3x + 1) + (x^2 - 2x - 3)}{(x-2)(x+1)(x-1)}$

$= \dfrac{3x^2 - 5x - 2}{(x-2)(x+1)(x-1)} = \dfrac{(3x+1)(x-2)}{(x-2)(x+1)(x-1)}$

if $x \neq 2$ $= \dfrac{(3x+1)}{(x+1)(x-1)}$

Section P.7 Exercises

1. $(-\infty, -9] \cup [1, \infty)$:

$x + 4 \geq 5$ or $x + 4 \leq -5$

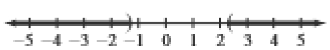
$x \geq 1$ or $x \leq -9$

2. $(-\infty, -1.3) \cup (2.3, \infty)$:

$2x - 1 > 3.6$ or $2x - 1 < -3.6$

$2x > 4.6$ or $2x < -2.6$

$x > 2.3$ or $x < -1.3$



3. $(1, 5)$: $-2 < x - 3 < 2$

$1 < x < 5$

4. $[-8, 2]$: $-5 \leq x + 3 \leq 5$

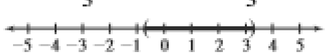
$-8 \leq x \leq 2$

5. $\left(-\dfrac{2}{3}, \dfrac{10}{3}\right)$: $|4 - 3x| < 6$

$-6 < 4 - 3x < 6$

$-10 < -3x < 2$

$\dfrac{10}{3} > x > -\dfrac{2}{3}$

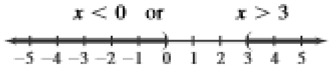


6. $(-\infty, 0) \cup (3, \infty)$: $|3 - 2x| > 3$

$3 - 2x > 3$ or $3 - 2x < -3$

$-2x > 0$ or $-2x < -6$

$x < 0$ or $x > 3$



7. $(-\infty, -11] \cup [7, \infty)$:

$\dfrac{x+2}{3} \leq -3$ or $\dfrac{x+2}{3} \geq 3$

$x + 2 \leq -9$ or $x + 2 \geq 9$

$x \leq -11$ or $x \geq 7$

8. $[-19, 29]$: $\left|\dfrac{x-5}{4}\right| \leq 6$

$-6 \leq \dfrac{x-5}{4} \leq 6$

$-24 \leq x - 5 \leq 24$

$-19 \leq x \leq 29$

9. $2x^2 + 17x + 21 = 0$

$(2x + 3)(x + 7) = 0$

$2x + 3 = 0$ or $x + 7 = 0$

$x = -\dfrac{3}{2}$ or $x = -7$

The graph of $y = 2x^2 + 17x + 21$ lies below the x-axis for $-7 < x < -\dfrac{3}{2}$. Hence $\left[-7, -\dfrac{3}{2}\right]$ is the solution since the endpoints are included.

10. $6x^2 - 13x + 6 = 0$

$(2x - 3)(3x - 2) = 0$

$2x - 3 = 0$ or $3x - 2 = 0$

$x = \dfrac{3}{2}$ or $x = \dfrac{2}{3}$

The graph of $y = 6x^2 - 13x + 6$ lies above the x-axis for $x < \dfrac{2}{3}$ and for $x > \dfrac{3}{2}$. Hence $\left(-\infty, \dfrac{2}{3}\right] \cup \left[\dfrac{3}{2}, \infty\right)$ is the solution since the endpoints are included.

11. $2x^2 + 7x - 15 = 0$

$(2x - 3)(x + 5) = 0$

$2x - 3 = 0$ or $x + 5 = 0$

$x = \dfrac{3}{2}$ or $x = -5$

The graph of $y = 2x^2 + 7x - 15$ lies above the x-axis for $x < -5$ and for $x > \dfrac{3}{2}$. Hence $(-\infty, -5) \cup \left(\dfrac{3}{2}, \infty\right)$ is the solution.

12. $4x^2 - 9x + 2 = 0$

$(4x - 1)(x - 2) = 0$

$4x - 1 = 0$ or $x - 2 = 0$

$x = \dfrac{1}{4}$ or $x = 2$

The graph of $y = 4x^2 - 9x + 2$ lies below the x-axis for $\dfrac{1}{4} < x < 2$. Hence $\left(\dfrac{1}{4}, 2\right)$ is the solution.

13. $2 - 5x - 3x^2 = 0$

$(2 + x)(1 - 3x) = 0$

$2 + x = 0$ or $1 - 3x = 0$

$x = -2$ or $x = \dfrac{1}{3}$

The graph of $y = 2 - 5x - 3x^2$ lies below the x-axis for $x < -2$ and for $x > \dfrac{1}{3}$. Hence $(-\infty, -2) \cup \left(\dfrac{1}{3}, \infty\right)$ is the solution.

14.
$$\begin{aligned} 21 + 4x - x^2 &= 0 \\ (7 - x)(3 + x) &= 0 \\ 7 - x = 0 \quad &\text{or} \quad 3 + x = 0 \\ x = 7 \quad &\text{or} \quad x = -3 \end{aligned}$$
The graph of $y = 21 + 4x - x^2$ lies above the x-axis for $-3 < x < 7$. Hence $(-3, 7)$ is the solution.

15.
$$\begin{aligned} x^3 - x &= 0 \\ x(x^2 - 1) &= 0 \\ x(x + 1)(x - 1) &= 0 \\ x = 0 \quad \text{or} \quad x + 1 = 0 \quad &\text{or} \quad x - 1 = 0 \\ x = 0 \quad \text{or} \quad x = -1 \quad &\text{or} \quad x = 1 \end{aligned}$$
The graph of $y = x^3 - x$ lies above the x-axis for $x > 1$ and for $-1 < x < 0$. Hence $[-1, 0] \cup [1, \infty)$ is the solution.

16.
$$\begin{aligned} x^3 - x^2 - 30x &= 0 \\ x(x^2 - x - 30) &= 0 \\ x(x - 6)(x + 5) &= 0 \\ x = 0 \quad \text{or} \quad x - 6 = 0 \quad &\text{or} \quad x + 5 = 0 \\ x = 0 \quad \text{or} \quad x = 6 \quad &\text{or} \quad x = -5 \end{aligned}$$
The graph of $y = x^3 - x^2 - 30x$ lies below the x-axis for $x < -5$ and for $0 < x < 6$. Hence $(-\infty, -5] \cup [0, 6]$ is the solution.

17. The graph of $y = x^2 - 4x - 1$ is zero for $x \approx -0.24$ and $x \approx 4.24$, and lies below the x-axis for $-0.24 < x < 4.24$. Hence $(-0.24, 4.24)$ is the approximate solution.

18. The graph of $y = 12x^2 - 25x + 12$ is zero for $x = \frac{4}{3}$ and $x = \frac{3}{4}$ and lies above the x-axis for $x < \frac{3}{4}$ and for $x > \frac{4}{3}$. Hence $\left(-\infty, \frac{3}{4}\right] \cup \left[\frac{4}{3}, \infty\right)$ is the solution.

19.
$$\begin{aligned} 6x^2 - 5x - 4 &= 0 \\ (3x - 4)(2x + 1) &= 0 \\ 3x - 4 = 0 \quad &\text{or} \quad 2x + 1 = 0 \\ x = \frac{4}{3} \quad &\text{or} \quad x = -\frac{1}{2} \end{aligned}$$
The graph of $y = 6x^2 - 5x - 4$ lies above the x-axis for $x < -\frac{1}{2}$ and for $x > \frac{4}{3}$. Hence $\left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{4}{3}, \infty\right)$ is the solution.

20.
$$\begin{aligned} 4x^2 - 1 &= 0 \\ (2x + 1)(2x - 1) &= 0 \\ 2x + 1 = 0 \quad &\text{or} \quad 2x - 1 = 0 \\ x = -\frac{1}{2} \quad &\text{or} \quad x = \frac{1}{2} \end{aligned}$$
The graph of $y = 4x^2 - 1$ lies below the x-axis for $-\frac{1}{2} < x < \frac{1}{2}$. Hence $\left[-\frac{1}{2}, \frac{1}{2}\right]$ is the solution.

21. The graph of $y = 9x^2 + 12x - 1$ appears to be zero for $x \approx -1.41$ and $x \approx 0.08$ and lies above the x-axis for $x < -1.41$ and $x > 0.08$. Hence $(-\infty, -1.41] \cup [0.08, \infty)$ is the approximate solution.

22. The graph of $y = 4x^2 - 12x + 7$ appears to be zero for $x \approx 0.79$ and $x \approx 2.21$ and lies below the x-axis for $0.79 < x < 2.21$. Hence $(0.79, 2.21)$ is the approximate solution.

23.
$$\begin{aligned} 4x^2 - 4x + 1 &= 0 \\ (2x - 1)(2x - 1) &= 0 \\ (2x - 1)^2 &= 0 \\ 2x - 1 &= 0 \\ x &= \frac{1}{2} \end{aligned}$$
The graph of $y = 4x^2 - 4x + 1$ lies entirely above the x-axis, except at $x = \frac{1}{2}$. Hence $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$ is the solution set.

24.
$$\begin{aligned} x^2 - 6x + 9 &= 0 \\ (x - 3)(x - 3) &= 0 \\ (x - 3)^2 &= 0 \\ x - 3 &= 0 \\ x &= 3 \end{aligned}$$
The graph of $y = x^2 - 6x + 9$ lies entirely above the x-axis, except at $x = 3$. Hence $x = 3$ is the only solution.

25.
$$\begin{aligned} x^2 - 8x + 16 &= 0 \\ (x - 4)(x - 4) &= 0 \\ (x - 4)^2 &= 0 \\ x - 4 &= 0 \\ x &= 4 \end{aligned}$$
The graph of $y = x^2 - 8x + 16$ lies entirely above the x-axis, except at $x = 4$. Hence there is no solution.

26.
$$\begin{aligned} 9x^2 + 12x + 4 &= 0 \\ (3x + 2)(3x + 2) &= 0 \\ (3x + 2)^2 &= 0 \\ 3x + 2 &= 0 \\ x &= -\frac{2}{3} \end{aligned}$$
The graph of $y = 9x^2 + 12x + 4$ lies entirely above the x-axis, except at $x = -\frac{2}{3}$. Hence every real number satisfies the inequality. The solution is $(-\infty, \infty)$.

27. The graph of $y = 3x^3 - 12x + 2$ is zero for $x \approx -2.08$, $x \approx 0.17$, and $x \approx 1.91$ and lies above the x-axis for $-2.08 < x < 0.17$ and $x > 1.91$. Hence, $[-2.08, 0.17] \cup [1.91, \infty)$ is the approximate solution.

28. The graph of $y = 8x - 2x^3 - 1$ is zero for $x \approx -2.06$, $x \approx 0.13$, and $x \approx 1.93$ and lies below the x-axis for $-2.06 < x < 0.13$ and $x > 1.93$. Hence, $(-2.06, 0.13) \cup (1.93, \infty)$ is the approximate solution.

29. $2x^3 + 2x > 5$ is equivalent to $2x^3 + 2x - 5 > 0$. The graph of $y = 2x^3 + 2x - 5$ is zero for $x \approx 1.11$ and lies above the x-axis for $x > 1.11$. So, $(1.11, \infty)$ is the approximate solution.

30. $4 \le 2x^3 + 8x$ is equivalent to $2x^3 + 8x - 4 \ge 0$. The graph of $y = 2x^3 + 8x - 4$ is zero for $x \approx 0.47$ and lies above the x-axis for $x > 0.47$. So, $[0.47, \infty)$ is the approximate solution.

31. Answers may vary. Here are some possibilities.

(a) $x^2 + 1 > 0$

(b) $x^2 + 1 < 0$

(c) $x^2 \le 0$

(d) $(x + 2)(x - 5) \le 0$

(e) $(x + 1)(x - 4) > 0$

(f) $x(x - 4) \ge 0$

32. $-16t^2 + 288t - 1152 = 0$
$t^2 - 18t + 72 = 0$
$(t - 6)(t - 12) = 0$
$t - 6 = 0$ or $t - 12 = 0$
$t = 6$ or $t = 12$
The graph of $-16t^2 + 288t - 1152$ lies above the t-axis for $6 < t < 12$. Hence $[6, 12]$ is the solution. This agrees with the result obtained in Example 10.

33. $s = -16t^2 + 256t$

(a) $-16t^2 + 256t = 768$
$-16t^2 + 256t - 768 = 0$
$t^2 - 16t + 48 = 0$
$(t - 12)(t - 4) = 0$
$t - 12 = 0$ or $t - 4 = 0$
$t = 12$ or $t = 4$
The projectile is 768 ft above ground twice: at $t = 4$ sec, on the way up, and $t = 12$ sec, on the way down.

(b) The graph of $s = -16t^2 + 256t$ lies above the graph of $s = 768$ for $4 < t < 12$. Hence the projectile's height will be at least 768 ft when t is in the interval $[4, 12]$.

(c) The graph of $s = -16t^2 + 256t$ lies below the graph of $s = 768$ for $0 < t < 4$ and $12 < t < 16$. Hence the projectile's height will be less than or equal to 768 ft when t is in the interval $(0, 4]$ or $[12, 16)$.

34. $s = -16t^2 + 272t$

(a) $-16t^2 + 272t = 960$
$-16t^2 + 272t - 960 = 0$
$t^2 - 17t + 60 = 0$
$(t - 12)(t - 5) = 0$
$t - 12 = 0$ or $t - 5 = 0$
$t = 12$ or $t = 5$
The projectile is 960 ft above ground twice: at $t = 5$ sec, on the way up, and $t = 12$ sec, on the way down.

(b) The graph of $s = -16t^2 + 272t$ lies above the graph of $s = 960$ for $5 < t < 12$. Hence the projectile's height will be more than 960 ft when t is in the interval $(5, 12)$.

(c) The graph of $s = -16t^2 + 272t$ lies below the graph of $s = 960$ for $0 < t < 5$ and $12 < t < 17$. Hence the projectile's height will be less than or equal to 960 ft when t is in the interval $(0, 5]$ or $[12, 17)$.

35. Solving the corresponding equation in the process of solving an inequality reveals the boundaries of the solution set. For example, to solve the inequality $x^2 - 4 \le 0$, we first solve the corresponding equation $x^2 - 4 = 0$ and find that $x = \pm 2$. The solution, $[-2, 2]$, of inequality has ± 2 as its boundaries.

36. Let x be her average speed; then $105 < 2x$. Solving this gives $x > 52.5$, so her least average speed is 52.5 mph.

37. (a) Let $x > 0$ be the width of a rectangle; then the length is $2x - 2$ and the perimeter is $P = 2[x + (2x - 2)]$. Solving $P < 200$ and $2x - 2 > 0$ gives 1 in. $< x <$ 34 in.
$2[x + (2x - 2)] < 200$ and $2x - 2 > 0$
$2(3x - 2) < 200$ $\quad$ $2x > 2$
$6x - 4 < 200$ $\quad$ $x > 1$
$6x < 204$
$x < 34$

(b) The area is $A = x(2x - 2)$. We already know $x > 1$ from **(a)**. Solve $A \le 1200$.
$x(2x - 2) = 1200$
$2x^2 - 2x - 1200 = 0$
$x^2 - x - 600 = 0$
$(x - 25)(x + 24) = 0$
$x - 25 = 0$ or $x + 24 = 0$
$x = 25$ or $x = -24$
The graph of $y = 2x^2 - 2x - 1200$ lies below the x-axis for $1 < x < 25$, so $A \le 1200$ when x is in the interval $(1, 25]$.

38. Substitute 20 and 40 into the equation $P = \dfrac{400}{V}$ to find the range for P: $P = \dfrac{400}{20} = 20$ and $P = \dfrac{400}{40} = 10$. The pressure can range from 10 to 20, or $10 \le P \le 20$.
Alternatively, solve graphically: graph $y = \dfrac{400}{x}$ on $[20, 40] \times [0, 30]$ and observe that all y-values are between 10 and 20.

39. Let x be the amount borrowed; then $\dfrac{200{,}000 + x}{50{,}000 + x} \ge 2$.
Solving for x reveals that the company can borrow no more than \$100,000.

40. False. If b is negative, there are no solutions, because the absolute value of a number is always nonnegative and every nonnegative real number is greater than any negative real number.

41. True. The absolute value of any real number is always nonnegative, i.e., greater than or equal to zero.

42. $|x - 2| < 3$
$-3 < x - 2 < 3$
$-1 < x < 5$
$(-1, 5)$
The answer is E.

43. The graph of $y = x^2 - 2x + 2$ lies entirely above the x-axis, so $x^2 - 2x + 2 \ge 0$ for all real numbers x. The answer is D.

44. $x^2 > x$ is true for all negative x, and for positive x when $x > 1$. So the solution is $(-\infty, 0) \cup (1, \infty)$. The answer is A.

45. $x^2 \le 1$ implies $-1 \le x \le 1$, so the solution is $[-1, 1]$. The answer is D.

46. (a) The lengths of the sides of the box are x, $12 - 2x$, and $15 - 2x$, so the volume is $x(12 - 2x)(15 - 2x)$. To solve $x(12 - 2x)(15 - 2x) = 125$, graph $y = x(12 - 2x)(15 - 2x)$ and $y = 125$ and find where the graphs intersect: Either $x \approx 0.94$ in or $x \approx 3.78$ in.

(b) The graph of $y = x(12 - 2x)(15 - 2x)$ lies above the graph of $y = 125$ for $0.94 < y < 3.78$ (approximately). So choosing x in the interval (0.94, 3.78) will yield a box with volume greater than 125 in^3.

(c) The graph of $y = x(12 - 2x)(15 - 2x)$ lies below the graph of $y = 125$ for $0 < y < 0.94$ and for $3.78 < x < 6$ (approximately). So choosing x in either interval (0, 0.94) or interval (3.78, 6) will yield a box with volume at most 125 in^3.