(d) Let $c = -1$. The graph suggests $y = -1$ does not intersect $y = |x^2 - 4|$. Since absolute value is never negative, $|x^2 - 4| = -1$ has no solutions.

(e) There is no other possible number of solutions of this equation. For any c, the solution involves solving two quadratic equations, each of which can have 0, 1, or 2 solutions.

70. (a) Let $D = b^2 - 4ac$. The two solutions are $\dfrac{-b \pm \sqrt{D}}{2a}$; adding them gives

$$\frac{-b+\sqrt{D}}{2a} + \frac{-b-\sqrt{D}}{2a} = \frac{-2b+\sqrt{D}-\sqrt{D}}{2a} = \frac{-2b}{2a} = -\frac{b}{a}$$

(b) Let $D = b^2 - 4ac$. The two solutions are $\dfrac{-b \pm \sqrt{D}}{2a}$; multiplying them gives

$$\frac{-b+\sqrt{D}}{2a} \cdot \frac{-b-\sqrt{D}}{2a} = \frac{(-b)^2 - (\sqrt{D})^2}{4a^2} = \frac{b^2 - (b^2 - 4ac)}{4a^2} = \frac{c}{a}$$

71. From #70(a), $x_1 + x_2 = -\dfrac{b}{a} = 5$. Since $a = 2$, this means $b = -10$. From #70(b), $x_1 \cdot x_2 = \dfrac{c}{a} = 3$; since $a = 2$, this means $c = 6$. The solutions are $\dfrac{10 \pm \sqrt{100 - 48}}{4}$; this reduces to $2.5 \pm \dfrac{1}{2}\sqrt{13}$, or approximately 0.697 and 4.303.

Section P.6 Complex Numbers

Quick Review P.6

1. $x + 9$
2. $x + 2y$
3. $a + 2d$
4. $5z - 4$
5. $x^2 - x - 6$
6. $2x^2 + 5x - 3$
7. $x^2 - 2$
8. $x^2 - 12$
9. $x^2 - 2x - 1$
10. $x^2 - 4x + 1$

Section P.6 Exercises

In #1–8, add or subtract the real and imaginary parts separately.

1. $(2 - 3i) + (6 + 5i) = (2 + 6) + (-3 + 5)i = 8 + 2i$

2. $(2 - 3i) + (3 - 4i) = (2 + 3) + (-3 - 4)i = 5 - 7i$

3. $(7 - 3i) + (6 - i) = (7 + 6) + (-3 - 1)i = 13 - 4i$

4. $(2 + i) - (9i - 3) = (2 + 3) + (1 - 9)i = 5 - 8i$

5. $(2 - i) + (3 - \sqrt{-3}) = (2 + 3) + (-1 - \sqrt{3})i$
$= 5 - (1 + \sqrt{3})i$

6. $(\sqrt{5} - 3i) + (-2 + \sqrt{-9}) = (\sqrt{5} - 2) + (-3 + 3)i$
$= (\sqrt{5} - 2) + 0i$

7. $(i^2 + 3) - (7 + i^3) = (-1 + 3) - (7 - i)$
$= (2 - 7) + i = -5 + i$

8. $(\sqrt{7} + i^2) - (6 - \sqrt{-81}) = (\sqrt{7} - 1)$
$- (6 - 9i) = (\sqrt{7} - 1 - 6) + 9i = (\sqrt{7} - 7) + 9i$

In #9–16, multiply out and simplify, recalling that $i^2 = -1$.

9. $(2 + 3i)(2 - i) = 4 - 2i + 6i - 3i^2$
$= 4 + 4i + 3 = 7 + 4i$

10. $(2 - i)(1 + 3i) = 2 + 6i - i - 3i^2$
$= 2 + 5i + 3 = 5 + 5i$

11. $(1 - 4i)(3 - 2i) = 3 - 2i - 12i + 8i^2$
$= 3 - 14i - 8 = -5 - 14i$

12. $(5i - 3)(2i + 1) = 10i^2 + 5i - 6i - 3$
$= -10 - i - 3 = -13 - i$

13. $(7i - 3)(2 + 6i) = 14i + 42i^2 - 6 - 18i$
$= -42 - 6 - 4i = -48 - 4i$

14. $(\sqrt{-4} + i)(6 - 5i) = (3i)(6 - 5i) = 18i - 15i^2$
$= 15 + 18i$

15. $(-3 - 4i)(1 + 2i) = -3 - 6i - 4i - 8i^2$
$= -3 - 10i + 8 = 5 - 10i$

16. $(\sqrt{-2} + 2i)(6 + 5i) = (\sqrt{2} + 2)i(6 + 5i)$
$= 6(2 + \sqrt{2})i + 5(2 + \sqrt{2})i^2$
$= -(10 + 5\sqrt{2}) + (12 + 6\sqrt{2})i$

17. $\sqrt{-16} = 4i$

18. $\sqrt{-25} = 5i$

19. $\sqrt{-3} = \sqrt{3}i$

20. $\sqrt{-5} = \sqrt{5}i$

In #21–24, equate the real and imaginary parts.

21. $x = 2, y = 3$

22. $x = 3, y = -7$

23. $x = 1, y = 2$

24. $x = 7, y = -7/2$

In #25–28, multiply out and simplify, recalling that $i^2 = -1$.

25. $(3 + 2i)^2 = 9 + 12i + 4i^2 = 5 + 12i$

26. $(1 - i)^3 = (1 - 2i + i^2)(1 - i) = (-2i)(1 - i)$
$= -2i + 2i^2 = -2 - 2i$

27. $\left(\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i\right)^4 = \left(\dfrac{\sqrt{2}}{2}\right)^4 (1 + i)^4$
$= \dfrac{1}{4}(1 + 2i + i^2)^2 = \dfrac{1}{4}(2i)^2 = \dfrac{1}{4}(-4) = -1 + 0i$

28. $\left(\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i\right)^3 = \left(\dfrac{1}{2}\right)^3 (\sqrt{3} + i)^3$
$= \dfrac{1}{8}(3 + 2\sqrt{3}\,i + i^2)(\sqrt{3} + i)$
$= \dfrac{1}{4}(1 + \sqrt{3}\,i)(\sqrt{3} + i)$
$= \dfrac{1}{4}(\sqrt{3} + i + 3i + \sqrt{3}\,i^2) = \dfrac{1}{4}(4i) = 0 + i$

In #29–32, recall that $(a + bi)(a - bi) = a^2 + b^2$.

29. $2^2 + 3^2 = 13$

30. $5^2 + 6^2 = 61$

31. $3^2 + 4^2 = 25$

32. $1^2 + (\sqrt{2})^2 = 3$

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In #33–40, multiply both the numerator and denominator by the complex conjugate of the denominator, recalling that $(a + bi)(a - bi) = a^2 + b^2$.

33. $\frac{1}{2+i} \cdot \frac{2-i}{2-i} = \frac{2-i}{5} = \frac{2}{5} - \frac{1}{5}i$

34. $\frac{i}{2-i} \cdot \frac{2+i}{2+i} = \frac{2i+i^2}{5} = -\frac{1}{5} + \frac{2}{5}i$

35. $\frac{2+i}{2-i} \cdot \frac{2+i}{2+i} = \frac{4+4i+i^2}{5} = \frac{3}{5} + \frac{4}{5}i$

36. $\frac{2+i}{3i} \cdot \frac{-3i}{-3i} = \frac{-6i-3i^2}{9} = \frac{1}{3} - \frac{2}{3}i$

37. $\frac{(2+i)^2(-i)}{1+i} \cdot \frac{1-i}{1-i} = \frac{(4+4i+i^2)(-i+i^2)}{2}$

$= \frac{(3+4i)(-1-i)}{2} = \frac{-3-3i-4i-4i^2}{2} = \frac{1}{2} - \frac{7}{2}i$

38. $\frac{(2-i)(1+2i)}{5+2i} \cdot \frac{5-2i}{5-2i} = \frac{(2+4i-i-2i^2)(5-2i)}{29}$

$= \frac{(4+3i)(5-2i)}{29} = \frac{20-8i+15i-6i^2}{29}$

$= \frac{26}{29} + \frac{7}{29}i$

39. $\frac{(1-i)(2-i)}{1-2i} \cdot \frac{1+2i}{1+2i} = \frac{(2-i-2i+i^2)(1+2i)}{5}$

$= \frac{(1-3i)(1+2i)}{5} = \frac{1+2i-3i-6i^2}{5} = \frac{7}{5} - \frac{1}{5}i$

40. $\frac{(1-\sqrt{2}i)(1+i)}{1+\sqrt{2}i} \cdot \frac{1-\sqrt{2}i}{1-\sqrt{2}i}$

$= \frac{(1+i-\sqrt{2}i-\sqrt{2}i^2)(1-\sqrt{2}i)}{3}$

$= \frac{[1+\sqrt{2}+(1-\sqrt{2})i](1-\sqrt{2}i)}{3}$

$= \frac{1+\sqrt{2}-(\sqrt{2}+2)i+(1-\sqrt{2})i-(\sqrt{2}-2)i^2}{3}$

$= \frac{1+\sqrt{2}+\sqrt{2}-2+(-2\sqrt{2}-1)i}{3}$

$= \frac{2\sqrt{2}-1}{3} - \frac{2\sqrt{2}+1}{3}i$

In #41–44, use the quadratic formula.

41. $x = -1 \pm 2i$

42. $x = -\frac{1}{6} \pm \frac{\sqrt{23}}{6}i$

43. $x = \frac{7}{8} \pm \frac{\sqrt{15}}{8}i$

44. $x = 2 \pm \sqrt{15}i$

45. False. When $a = 0$, $z = a + bi$ becomes $z = bi$, and then $-\bar{z} = -(-bi) = bi = z$.

46. True. Because $i^2 = -1$, $i^3 = i(i^2) = -i$, and $i^4 = (i^2)^2 = 1$, we obtain $i + i^2 + i^3 + i^4 = i + (-1) + (-i) + 1 = 0$.

47. $(2 + 3i)(2 - 3i)$ is a product of conjugates and equals $2^2 + 3^2 = 13 + 0i$. The answer is E.

48. $\frac{1}{i} = \frac{1}{i} \cdot \frac{-i}{-i} = \frac{-i}{1} = -1 + 0i$. The answer is E.

49. Complex, nonreal solutions of polynomials with real coefficients always come in conjugate pairs. So another solution is $2 + 3i$, and the answer is A.

50. $(1 - i)^3 = (-2i)(1 - i) = -2i + 2i^2 = -2 - 2i$. The answer is C.

51. (a) $i = i$ $\quad i^5 = i \cdot i^4 = i$
$i^2 = -1$ $\quad i^6 = i^2 \cdot i^4 = -1$
$i^3 = (-1)i = -i$ $\quad i^7 = i^3 \cdot i^4 = -i$
$i^4 = (-1)^2 = 1$ $\quad i^8 = i^4 \cdot i^4 = 1 \cdot 1 = 1$

(b) $i^{-1} = \frac{1}{i} = \frac{1}{i} \cdot \frac{i}{i} = -i$ $\quad i^{-5} = \frac{1}{i} \cdot \frac{1}{i^4} = \frac{1}{i} = -i$

$i^{-2} = \frac{1}{i^2} = -1$ $\quad i^{-6} = \frac{1}{i^2} \cdot \frac{1}{i^4} = -1$

$i^{-3} = \frac{1}{i} \cdot \frac{1}{i^2} = -\frac{1}{i} = i$ $\quad i^{-7} = \frac{1}{i^3} \cdot \frac{1}{i^4} = -\frac{1}{i} = i$

$i^{-4} = \frac{1}{i^2} \cdot \frac{1}{i^2} = (-1)(-1) = 1$ $\quad i^{-8} = \frac{1}{i^4} \cdot \frac{1}{i^4} = 1 \cdot 1 = 1$

(c) $i^0 = 1$

(d) Answers will vary.

52. Answers will vary. One possibility: The graph has the shape of a parabola, but does not cross the x-axis when plotted in the real plane, beacuse it does not have any real zeros. As a result, the function will *always* be positive or *always* be negative.

53. Let a and b be any two real numbers. Then $(a + bi) - (a - bi) = (a - a) + (b + b)i = 0 + 2bi = 2bi$.

54. $(a + bi)\overline{(a + bi)} = (a + bi)(a - bi) = a^2 + b^2$, imaginary part is zero.

55. $\overline{(a + bi)(c + di)} = \overline{(ac - bd) + (ad + bc)i} = (ac - bd) - (ad + bc)i$ and $\overline{(a + bi)}\,\overline{(c + di)} = (a - bi)\cdot(c - di) = (ac - bd) - (ad + bc)i$ are equal.

56. $\overline{(a + bi) + (c + di)} = \overline{(a + c) + (b + d)i} = (a + c) - (b + d)i$ and $\overline{(a + bi)} + \overline{(c + di)} = (a - bi) + (c - di) = (a + c) - (b + d)i$ are equal.

57. $(-i)^2 - i(-i) + 2 = 0$ but $(i)^2 - i(i) + 2 \neq 0$. Because the coefficient of x in $x^2 - ix + 2 = 0$ is not a real number, the complex conjugate, i, of $-i$, need not be a solution.

Section P.7 Solving Inequalities Algebraically and Graphically

Quick Review P.7

1. $-7 < 2x - 3 < 7$
$-4 < 2x < 10$
$-2 < x < 5$

2. $5x - 2 \geq 7x + 4$
$-2x \geq 6$
$x \leq -3$

3. $|x + 2| = 3$
$x + 2 = 3$ or $x + 2 = -3$
$x = 1$ or $x = -5$

4. $4x^2 - 9 = (2x - 3)(2x + 3)$

5. $x^3 - 4x = x(x^2 - 4) = x(x - 2)(x + 2)$

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