AP Calculus AB: Section I, Part A

50 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_1^2 (4x^3 - 6x)\, dx =$

$x^4 - 3x^2 \Big|_1^2$

$(16-12)-(1-3)$

$4-(-2)$ 6

(A) 2
(B) 4
(C) 6
(D) 36
(E) 42

2. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

$f'(x) = x\left(\frac{1}{2}(2x-3)^{-1/2} \cdot 2\right) + \sqrt{2x-3}$

$= \frac{x}{\sqrt{2x-3}} + \frac{2x-3}{\sqrt{2x-3}}$

$= \frac{3x-3}{\sqrt{2x-3}}$

(A) $\frac{3x-3}{\sqrt{2x-3}}$

(B) $\frac{x}{\sqrt{2x-3}}$

(C) $\frac{1}{\sqrt{2x-3}}$

(D) $\frac{-x+3}{\sqrt{2x-3}}$

(E) $\frac{5x-6}{2\sqrt{2x-3}}$

3. If $\int_a^b f(x)\, dx = a + 2b$, then $\int_a^b (f(x)+5)\, dx =$

$\int_a^b f(x)\, dx + \int_a^b 5\, dx$ $a+2b+5b-5a$

$-4a+7b$

(A) $a+2b+5$ (B) $5b-5a$ (C) $7b-4a$ (D) $7b-5a$ (E) $7b-6a$

4. If $f(x) = -x^3 + x + \frac{1}{x}$, then $f'(-1) =$

$f'(-1) = -3(-1)^2 + 1 - \frac{1}{(-1)^2} = -3+1-1$

(A) 3 (B) 1 (C) -1 (D) -3 (E) -5

5. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

(A) $x < 0$

(B) $x > 0$

(C) $x < -2$ or $x > -\frac{2}{3}$

(D) $x < \frac{2}{3}$ or $x > 2$

(E) $\frac{2}{3} < x < 2$

$y' = 12x^3 - 48x^2 + 48x$

$y'' = 36x^2 - 96x + 48$

$0 = 12(3x^2 - 8x + 4)$

$= 12(3x-2)(x-2)$

$- -$ $+ -$ $+ +$

P   2/3   N   2   P

6. $\frac{1}{2}\int e^{\frac{t}{2}}\,dt =$

$u = \frac{1}{2}t$

$du = \frac{1}{2}dt$ $\int e^u\,du = e^u + C$

(A) $e^{-t} + C$ (B) $e^{-\frac{t}{2}} + C$ (C) $e^{\frac{t}{2}} + C$ (D) $2e^{\frac{t}{2}} + C$ (E) $e^{t} + C$

7. $\frac{d}{dx}\cos^2(x^3) =$

$\frac{d}{dx}(\cos(x^3))^2$

$= 2(\cos x^3)\cdot(-\sin x^3)\cdot 3x^2$

$= -6x^2\cos(x^3)\sin(x^3)$

(A) $6x^2\sin(x^3)\cos(x^3)$

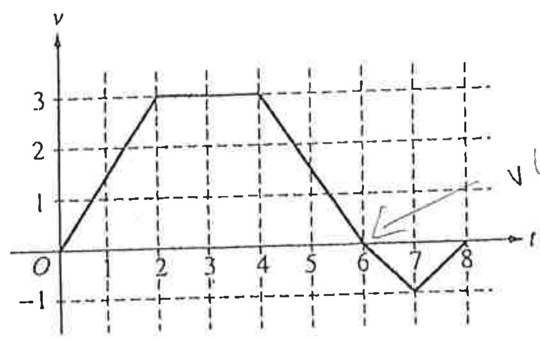
(B) $6x^2\cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2\sin(x^3)\cos(x^3)$

(E) $-2\sin(x^3)\cos(x^3)$

Questions 8-9 refer to the following situation.



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

8. At what value of t does the bug change direction?

(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

9. What is the total distance the bug traveled from $t = 0$ to $t = 8$?

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6

10. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

(A) $y - 1 = -\left(x - \frac{\pi}{4}\right)$

(B) $y - 1 = -2\left(x - \frac{\pi}{4}\right)$

(C) $y = 2\left(x - \frac{\pi}{4}\right)$

(D) $y = -\left(x - \frac{\pi}{4}\right)$

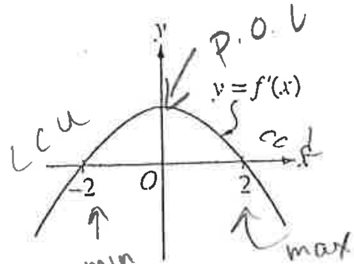
(E) $y = -2\left(x - \frac{\pi}{4}\right)$

$y' = -\sin(2x) \cdot 2$

$= -2\sin\left(2\left(\frac{\pi}{4}\right)\right)$

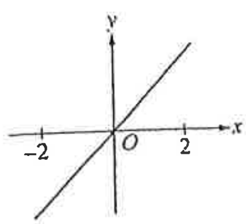
$= -2\sin\frac{\pi}{2} = -2$ $m = -2$

$y = \cos\frac{\pi}{2} = 0$ $\left(\frac{\pi}{4}, 0\right)$

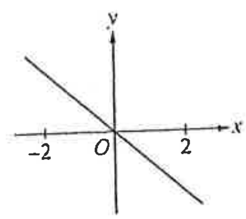


11. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

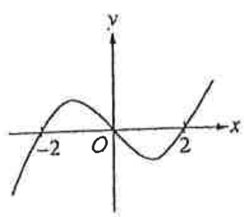
(A)



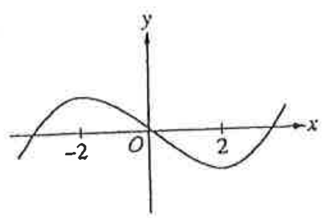
(B)



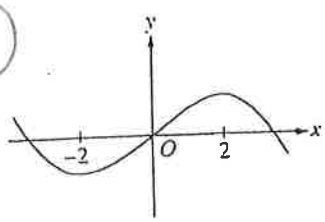
(C)



(D)



(E)



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12. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

$-4y = -2x + 3$

$y = \frac{1}{2}x - \frac{3}{4}$

(A) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{1}{8}\right)$ (C) $\left(1, -\frac{1}{4}\right)$ (D) $\left(1, \frac{1}{2}\right)$ (E) $(2, 2)$

$y' = x$

$\frac{1}{2} = x$

$y = \frac{1}{2}\left(\frac{1}{2}\right)^2 = \frac{1}{2}\left(\frac{1}{4}\right) = \frac{1}{8}$

13. Let f be a function defined for all real numbers x. If $f'(x) = \frac{|4-x^2|}{x-2}$, then f is decreasing on the interval

vo when $x-2<0$
$x<2$

(A) $(-\infty, 2)$ (B) $(-\infty, \infty)$ (C) $(-2, 4)$ (D) $(-2, \infty)$ (E) $(2, \infty)$

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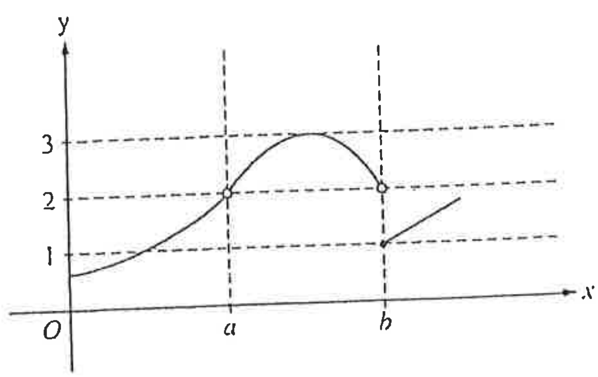
14. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find an approximation to a zero of f, that approximation is

$y-2=5(x-3)$
$0-2=5x-15$
$13=5x$

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5



15. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$

(B) $\lim_{x \to a} f(x) = 2$

(C) $\lim_{x \to b} f(x) = 2$

(D) $\lim_{x \to b} f(x) = 1$

(E) $\lim_{x \to a} f(x)$ does not exist.

$\int_{-2}^{2} 5-(x^2+1)\,dx = \int_{-2}^{2} (4-x^2)\,dx = 4x-\frac{x^3}{3}\Big|_{-2}^{2}$ $\quad 8-\frac{8}{3} + 8-\frac{8}{3}$

$16-\frac{16}{3}$

$\frac{48}{3}-\frac{16}{3}$

$\frac{32}{3}$

$5=x^2+1$

$x=\pm 2$

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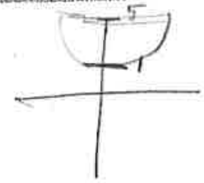


16. The area of the region enclosed by the graph of $y=x^2+1$ and the line $y=5$ is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$ (D) $\frac{32}{3}$ (E) 8π

17. If $x^2+y^2=25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4,3)$?

4/4

(A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

18. $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}\,dx$ is $\quad \int_0^{\frac{\pi}{4}} \sec^2 x\, e^{\tan x}\,dx \quad u=\tan x \quad du=\sec^2 x\,dx \quad u(\pi/4)=1 \quad u(0)=0$

4/4

(A) 0 (B) 1 (C) $e-1$ (D) e (E) $e+1$ $\quad \int_0^1 e^u\,du = e^1-e^0 = e-1$

19. If $f(x)=\ln\left|x^2-1\right|$, then $f'(x)=$ $\quad \frac{1}{x^2-1}(2x) = \frac{2x}{x^2-1}$

(A) $\left|\frac{2x}{x^2-1}\right|$

oops (B) $\frac{2x}{\left|x^2-1\right|}$

(C) $\frac{2|x|}{x^2-1}$

(D) $\frac{2x}{x^2-1}$

(E) $\frac{1}{x^2-1}$

$\sin(-x) = -\sin x$

20. The average value of $\cos x$ on the interval $[-3,5]$ is

(A) $\dfrac{\sin 5-\sin 3}{8}$

(B) $\dfrac{\sin 5-\sin 3}{2}$

(C) $\dfrac{\sin 3-\sin 5}{2}$

(D) $\dfrac{\sin 3+\sin 5}{2}$

(E) $\dfrac{\sin 3+\sin 5}{8}$

$$\frac{1}{5-(-3)}\int_{-3}^{5} \cos x\,dx = \frac{1}{8}\sin x\Big|_{-3}^{5}$$

$$= \frac{\sin 5-(\sin(-3))}{8}$$

$$= \frac{\sin 5+\sin 3}{8}$$

odd function

21. $\lim\limits_{x\to 1}\dfrac{x}{\ln x}$ is

(A) 0 (B) $\dfrac{1}{e}$ (C) 1 (D) e (E) nonexistent

22. What are all values of x for which the function f defined by $f(x)=(x^2-3)e^{-x}$ is increasing?

(A) There are no such values of x.

(B) $x<-1$ and $x>3$

(C) $-3<x<1$

(D) $-1<x<3$

(E) All values of x

$$f'(x) = (x^2-3)(-e^{-x}) + e^{-x}(2x)$$

$$= e^{-x}(-x^2+3+2x)$$

$$= -e^{-x}(x^2-2x-3)$$

$(x-3)(x+1)$

$-+\quad --\quad -+$

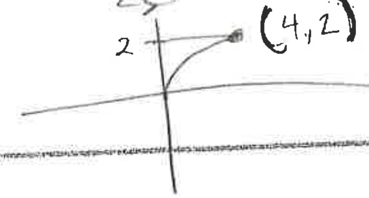
N -1 P 3 N

23. If the region enclosed by the y-axis, the line $y=2$, and the curve $y=\sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

(A) $\dfrac{32\pi}{5}$ (B) $\dfrac{16\pi}{3}$ (C) $\dfrac{16\pi}{5}$ (D) $\dfrac{8\pi}{3}$ (E) π

$x=y^2$ $r=y^2$

$(4,2)$ $A_{cs}=(y^2)^2\pi$



$$\pi\int_0^2 y^4\,dy \quad \frac{y^5}{5}\pi\Big|_0^2 = \frac{32\pi}{5}$$

24. The expression $\frac{1}{50}\left(\sqrt{\frac{1}{50}}+\sqrt{\frac{2}{50}}+\sqrt{\frac{3}{50}}+\cdots+\sqrt{\frac{50}{50}}\right)$ is a Riemann sum approximation for

interval 0, 1

$f(x)=\sqrt{x}$

$\frac{1-0}{50}=\frac{1}{50}$

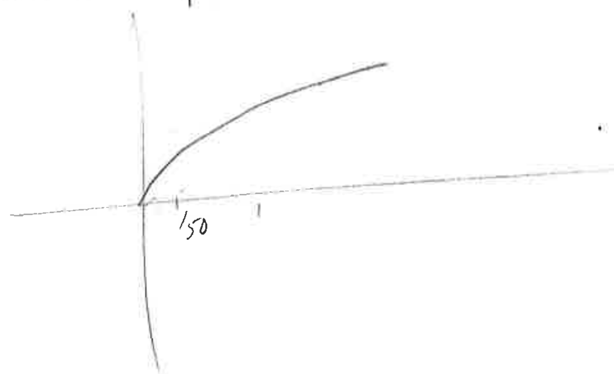
(A) $\int_0^1 \sqrt{\frac{x}{50}}\,dx$

(B) $\int_0^1 \sqrt{x}\,dx$

(C) $\frac{1}{50}\int_0^1 \sqrt{\frac{x}{50}}\,dx$

(D) $\frac{1}{50}\int_0^1 \sqrt{x}\,dx$

(E) $\frac{1}{50}\int_0^{50} \sqrt{x}\,dx$



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25. $\int x\sin(2x)\,dx=$

$x \cdot 2\sin x\cos x$

$2\ x \sin x \cos x$

(A) $-\frac{x}{2}\cos(2x)+\frac{1}{4}\sin(2x)+C$

(B) $-\frac{x}{2}\cos(2x)-\frac{1}{4}\sin(2x)+C$

(C) $\frac{x}{2}\cos(2x)-\frac{1}{4}\sin(2x)+C$

(D) $-2x\cos(2x)+\sin(2x)+C$

(E) $-2x\cos(2x)-4\sin(2x)+C$

Don't assign

must do

by parts