

NO Calculator!

Directions: Show work directly on this page. Circle your answer. Answer every problem.

1. What is the x -coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

(A) 5

(B) 0

(C) $-\frac{10}{3}$

(D) -5

(E) -10

$$y' = x^2 + 10x$$

$$y'' = 2x + 10 = 0$$

$$2x = -10$$

$$x = -5$$

$$y'' \quad \begin{array}{c} | \\ -5 \end{array}$$

$$2. \int_1^2 \frac{1}{x^2} dx = \int_1^2 x^{-2} dx = -x^{-1} \Big|_1^2 = -\frac{1}{x} \Big|_1^2 = -\frac{1}{2} - \left(-\frac{1}{1}\right) = \frac{1}{2}$$

(A) $-\frac{1}{2}$ (B) $\frac{7}{24}$ (C) $\frac{1}{2}$

(D) 1

(E) $2 \ln 2$ 3. If f is continuous for $a \leq x \leq b$ and differentiable for $a < x < b$, which of the following could be false?(A) $f'(c) = \frac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$. **MVT**(B) $f'(c) = 0$ for some c such that $a < c < b$.(C) f has a minimum value on $a \leq x \leq b$.(D) f has a maximum value on $a \leq x \leq b$.(E) $\int_a^b f(x) dx$ exists. **FTC**> **EVT**

- Product Rule
 4. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

$$2^2 + 2y = 10 \quad (2, 3)$$

$$2y = 6 \quad y = 3$$

(A) $-\frac{7}{2}$

(B) -2

(C) $\frac{2}{7}$

(D) $\frac{3}{2}$

(E) $\frac{7}{2}$

$$2x + y + x \frac{dy}{dx} = 0$$

$$x \frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x - y}{x} \quad \left. \frac{dy}{dx} \right|_{(2,3)} = \frac{-2(2) - (3)}{2} = -\frac{7}{2}$$

5. $\int_1^e \left(\frac{x^2 - 1}{x} \right) dx =$

(A) $e - \frac{1}{e}$

(B) $e^2 - e$

(C) $\frac{e^2}{2} - e + \frac{1}{2}$

(D) $e^2 - 2$

(E) $\frac{e^2}{2} - \frac{3}{2}$

$$\int_1^e \left(x - \frac{1}{x} \right) dx = \left(\frac{1}{2}x^2 - \ln x \right) \Big|_1^e$$

$$= \frac{1}{2}e^2 - \ln e - \left(\frac{1}{2}(1)^2 - \ln 1 \right)$$

$$= \frac{1}{2}e^2 - 1 - \frac{1}{2} + 0$$

$$= \frac{e^2}{2} - \frac{3}{2}$$

6. Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x

(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

(A) $f'(x)$

Product rule

(B) $g(x)$

(C) e^x

(D) 0

(E) 1

$$h'(x) = f'(x)g(x) + f(x)g'(x)$$

$$\text{Since } h'(x) = f(x)g'(x), \quad f'(x) = 0$$

$$\text{Since } f(0) = 1, \quad f(x) \neq 0$$

7. If $F(x) = \int_0^x \sqrt{t^3 + 1} dt$, then $F'(2) =$

(A) -3

(B) -2

(C) 2

(D) 3

(E) 18

$$F'(x) = \sqrt{x^3 + 1} \quad \text{FTC}$$

$$F'(2) = \sqrt{2^3 + 1}$$

$$= \sqrt{9}$$

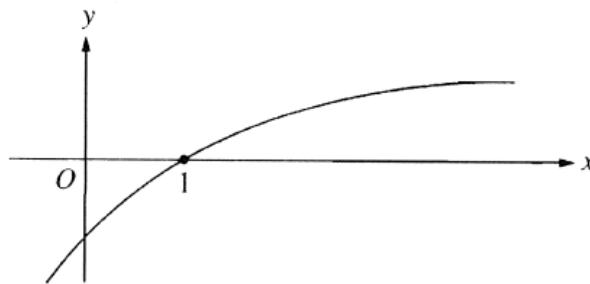
8. If $f(x) = \sin(e^{-x})$, then $f'(x) =$

- (A) $-\cos(e^{-x})$
- (B) $\cos(e^{-x}) + e^{-x}$
- (C) $\cos(e^{-x}) - e^{-x}$
- (D) $e^{-x} \cos(e^{-x})$
- (E) $-e^{-x} \cos(e^{-x})$

Chain Rule

$$f'(x) = \cos(e^{-x}) \cdot -e^{-x} = -e^{-x} \cos(e^{-x})$$

9.



The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

- (A) $f(1) < f'(1) < f''(1)$
- (B) $f(1) < f''(1) < f'(1)$
- (C) $f'(1) < f(1) < f''(1)$
- (D) $f''(1) < f(1) < f'(1)$
- (E) $f''(1) < f'(1) < f(1)$

$$f(1) = 0$$

$$f'(1) > 0$$

$$f''(1) < 0 \text{ (c.d.)}$$

sign change of f''

10. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

- (A) -1 only
- (B) 2 only
- (C) -1 and 0 only
- (D) -1 and 2 only
- (E) -1, 0, and 2 only

