MC Solutions

Thursday, April 13, 2017 10:39 AM

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1997 Calculus AB Solutions: Part B

76. E $f(x)=\dfrac{e^{2x}}{2x}$; $f'(x)=\dfrac{2e^{2x}\cdot 2x-2e^{2x}}{4x^2}=\dfrac{e^{2x}(2x-1)}{2x^2}$

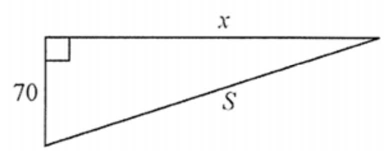
77. D $y=x^3+6x^2+7x-2\cos x$. Look at the graph of $y''=6x+12+2\cos x$ in the window $[-3,-1]$ since that domain contains all the option values. y'' changes sign at $x=-1.89$.

78. D $F(3)-F(0)=\int_0^3 f(x)dx=\int_0^1 f(x)dx+\int_1^3 f(x)dx=2+2.3=4.3$
(Count squares for $\int_0^1 f(x)dx$)

79. C The stem of the questions means $f'(2)=5$. Thus f is differentiable at $x=2$ and therefore continuous at $x=2$. We know nothing of the continuity of f'. I and II only.

80. A $f(x)=2e^{4x^2}$; $f'(x)=16xe^{4x^2}$; We want $16xe^{4x^2}=3$. Graph the derivative function and the function $y=3$, then find the intersection to get $x=0.168$.

81. A Let x be the distance of the train from the crossing. Then $\dfrac{dx}{dt}=60$.
$S^2=x^2+70^2 \Rightarrow 2S\dfrac{dS}{dt}=2x\dfrac{dx}{dt} \Rightarrow \dfrac{dS}{dt}=\dfrac{x}{S}\dfrac{dx}{dt}$.
After 4 seconds, $x=240$ and so $S=250$.
Therefore $\dfrac{dS}{dt}=\dfrac{240}{250}(60)=57.6$



82. B $P(x)=2x^2-8x$; $P'(x)=4x-8$; P' changes from negative to positive at $x=2$. $P(2)=-8$

83. C $\cos x=x$ at $x=0.739085$. Store this in A. $\int_0^A(\cos x-x)\,dx=0.400$

84. C Cross sections are squares with sides of length y.
Volume $=\int_1^e y^2dx=\int_1^e \ln x\,dx=(x\ln x-x)\Big|_1^e=(e\ln e-e)-(0-1)=1$

85. C Look at the graph of f' and locate where the y changes from positive to negative. $x=0.91$

86. A $f(x)=\sqrt{x}$; $f'(x)=\dfrac{1}{2\sqrt{x}}$; $\dfrac{1}{2\sqrt{c}}=2\cdot\dfrac{1}{2\sqrt{1}} \Rightarrow c=\dfrac{1}{4}$

87. B $a(t) = t + \sin t$ and $v(0) = -2 \Rightarrow v(t) = \frac{1}{2}t^2 - \cos t - 1$; $v(t) = 0$ at $t = 1.48$

88. E $f(x) = \int_a^x h(x)dx \Rightarrow f(a) = 0$, therefore only (A) or (E) are possible. But $f'(x) = h(x)$ and therefore f is differentiable at $x = b$. This is true for the graph in option (E) but not in option (A) where there appears to be a corner in the graph at $x = b$. Also, Since h is increasing at first, the graph of f must start out concave up. This is also true in (E) but not (A).

89. B $T = \frac{1}{2} \cdot \frac{1}{2}(3 + 2 \cdot 3 + 2 \cdot 5 + 2 \cdot 8 + 13) = 12$

90. D

$F(x) = \frac{1}{2}\sin^2 x$	$F'(x) = \sin x \cos x$	Yes
$F(x) = \frac{1}{2}\cos^2 x$	$F'(x) = -\cos x \sin x$	No
$F(x) = -\frac{1}{4}\cos(2x)$	$F'(x) = \frac{1}{2}\sin(2x) = \sin x \cos x$	Yes

1998 Calculus AB Solutions: Part A

1. D $y' = x^2 + 10x$; $y'' = 2x + 10$; y'' changes sign at $x = -5$

2. B $\int_{-1}^{4} f(x)dx = \int_{-1}^{2} f(x)dx + \int_{2}^{4} f(x)dx$

   $= \text{Area of trapezoid}(1) - \text{Area of trapezoid}(2) = 4 - 1.5 = 2.5$

3. C $\int_1^2 \frac{1}{x^2}dx = \int_1^2 x^{-2}dx = -x^{-1}\Big|_1^2 = \frac{1}{2}$

4. B This would be false if f was a linear function with non-zero slope.

5. E $\int_0^x \sin t\,dt = -\cos t\big|_0^x = -\cos x - (-\cos 0) = -\cos x + 1 = 1 - \cos x$

6. A Substitute $x = 2$ into the equation to find $y = 3$. Taking the derivative implicitly gives $\frac{d}{dx}\left(x^2 + xy\right) = 2x + xy' + y = 0$. Substitute for x and y and solve for y'.

   $4 + 2y' + 3 = 0;\ y' = -\frac{7}{2}$

7. E $\int_1^e \frac{x^2-1}{x}\,dx = \int_1^e x - \frac{1}{x}\,dx = \left(\frac{1}{2}x^2 - \ln x\right)\Bigg|_1^e = \left(\frac{1}{2}e^2 - 1\right) - \left(\frac{1}{2} - 0\right) = \frac{1}{2}e^2 - \frac{3}{2}$

8. E $h(x) = f(x)g(x)$ so, $h'(x) = f'(x)g(x) + f(x)g'(x)$. It is given that $h'(x) = f(x)g'(x)$. Thus, $f'(x)g(x) = 0$. Since $g(x) > 0$ for all x, $f'(x) = 0$. This means that f is constant. It is given that $f(0) = 1$, therefore $f(x) = 1$.

9. D Let $r(t)$ be the rate of oil flow as given by the graph, where t is measured in hours. The total number of barrels is given by $\int_0^{24} r(t)dt$. This can be approximated by counting the squares below the curve and above the horizontal axis. There are approximately five squares with area 600 barrels. Thus the total is about $3,000$ barrels.

10. D $f'(x) = \frac{(x-1)(2x) - (x^2-2)(1)}{(x-1)^2}$; $f'(2) = \frac{(2-1)(4) - (4-2)(1)}{(2-1)^2} = 2$

11. A Since f is linear, its second derivative is zero. The integral gives the area of a rectangle with zero height and width $(b-a)$. This area is zero.

12. E $\lim_{x\to 2^-} f(x) = \ln 2 \neq 4\ln 2 = \lim_{x\to 2^+} f(x)$. Therefore the limit does not exist.

13. B At $x = 0$ and $x = 2$ only. The graph has a non-vertical tangent line at every other point in the interval and so has a derivative at each of these other x's.

14. C $v(t) = 2t - 6$; $v(t) = 0$ for $t = 3$

15 D By the Fundamental Theorem of Calculus, $F'(x) = \sqrt{x^3+1}$, thus $F'(2) = \sqrt{2^3+1} = \sqrt{9} = 3$.

16. E $f'(x) = \cos(e^{-x})\cdot\frac{d}{dx}(e^{-x}) = \cos(e^{-x})\left(e^{-x}\cdot\frac{d}{dx}(-x)\right) = -e^{-x}\cos(e^{-x})$

17. D From the graph $f(1) = 0$. Since $f'(1)$ represents the slope of the graph at $x = 1$, $f'(1) > 0$. Also, since $f''(1)$ represents the concavity of the graph at $x = 1$, $f''(1) < 0$.

18. B $y' = 1 - \sin x$ so $y'(0) = 1$ and the line with slope 1 containing the point $(0,1)$ is $y = x + 1$.

19. C Points of inflection occur where f'' changes sign. This is only at $x = 0$ and $x = -1$. There is no sign change at $x = 2$.

20. A $\int_{-3}^{k} x^2\,dx = \frac{1}{3}x^3\Big|_{-3}^{k} = \frac{1}{3}\left(k^3 - (-3)^3\right) = \frac{1}{3}\left(k^3 + 27\right) = 0$ only when $k = -3$.

21. B The solution to this differential equation is known to be of the form $y = y(0)\cdot e^{kt}$. Option (B) is the only one of this form. If you do not remember the form of the solution, then separate the variables and antidifferentiate.
$\frac{dy}{y} = k\,dt$; $\ln|y| = kt + c_1$; $|y| = e^{kt+c_1} = e^{kt}e^{c_1}$; $y = ce^{kt}$.

22. C f is increasing on any interval where $f'(x) > 0$. $f'(x) = 4x^3 + 2x = 2x(2x^2+1) > 0$. Since $(x^2+1) > 0$ for all x, $f'(x) > 0$ whenever $x > 0$.

23. A The graph shows that f is increasing on an interval (a,c) and decreasing on the interval (c,b), where $a < c < b$. This means the graph of the derivative of f is positive on the interval (a,c) and negative on the interval (c,b), so the answer is (A) or (E). The derivative is not (E), however, since then the graph of f would be concave down for the entire interval.

24. D The maximum acceleration will occur when its derivative changes from positive to negative or at an endpoint of the interval. $a(t) = v'(t) = 3t^2 - 6t + 12 = 3(t^2 - 2t + 4)$ which is always positive. Thus the acceleration is always increasing. The maximum must occur at $t = 3$ where $a(3) = 21$

25. D The area is given by $\int_0^2 x^2 - (-x)\,dx = \left(\frac{1}{3}x^3 + \frac{1}{2}x^2\right)\Big|_0^2 = \frac{8}{3} + 2 = \frac{14}{3}$.

26. A Any value of k less than 1/2 will require the function to assume the value of 1/2 at least twice because of the Intermediate Value Theorem on the intervals [0,1] and [1,2]. Hence $k = 0$ is the only option.

27. A $\frac{1}{2}\int_0^2 x^2\sqrt{x^3+1}\,dx = \frac{1}{2}\int_0^2 (x^3+1)^{\frac{1}{2}}\left(\frac{1}{3}\cdot 3x^2\right)dx = \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{2}{3}(x^3+1)^{\frac{3}{2}}\Big|_0^2 = \frac{1}{9}\left(9^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) = \frac{26}{9}$

28. E $f'(x) = \sec^2(2x)\cdot\frac{d}{dx}(2x) = 2\sec^2(2x)$; $f'\left(\frac{\pi}{6}\right) = 2\sec^2\left(\frac{\pi}{3}\right) = 2(4) = 8$