

MC MC

1993 Calculus AB Solutions

1. C $f'(x) = \frac{3}{2}x^{\frac{1}{2}}; f'(4) = \frac{3}{2} \cdot 4^{\frac{1}{2}} = \frac{3}{2} \cdot 2 = 3$

2. B Summing pieces of the form: (vertical) · (small width), vertical = $(d - f(x))$, width = Δx
 Area = $\int_a^b (d - f(x)) dx$

3. D Divide each term by n^3 . $\lim_{n \rightarrow \infty} \frac{3n^3 - 5n}{n^3 - 2n^2 + 1} = \lim_{n \rightarrow \infty} \frac{3 - \frac{5}{n^2}}{1 - \frac{2}{n} + \frac{1}{n^3}} = 3$

4. A $3x^2 + 3(y + x \cdot y') + 6y^2 \cdot y' = 0; y'(3x + 6y^2) = -(3x^2 + 3y)$
 $y' = -\frac{3x^2 + 3y}{3x + 6y^2} = -\frac{x^2 + y}{x + 2y^2}$

5. A $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x + 2} = \lim_{x \rightarrow -2} \frac{(x + 2)(x - 2)}{x + 2} = \lim_{x \rightarrow -2} (x - 2) = -4$. For continuity $f(-2)$ must be -4 .

6. D Area = $\int_3^4 \frac{1}{x-1} dx = (\ln|x-1|) \Big|_3^4 = \ln 3 - \ln 2 = \ln \frac{3}{2}$

7. B $y' = \frac{2 \cdot (3x-2) - (2x+3) \cdot 3}{(3x-2)^2}; y'(1) = -13$. Tangent line: $y - 5 = -13(x - 1) \Rightarrow 13x + y = 18$

8. E $y' = \sec^2 x + \csc^2 x$

9. E $h(x) = f(|x|) = 3|x|^2 - 1 = 3x^2 - 1$

10. D $f'(x) = 2(x-1) \cdot \sin x + (x-1)^2 \cos x; f'(0) = (-2) \cdot 0 + 1 \cdot 1 = 1$

11. C $a(t) = 6t - 2; v(t) = 3t^2 - 2t + C$ and $v(3) = 25 \Rightarrow 25 = 27 - 6 + C; v(t) = 3t^2 - 2t + 4$
 $x(t) = t^3 - t^2 + 4t + K$; Since $x(1) = 10, K = 6; x(t) = t^3 - t^2 + 4t + 6$.

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12. B The only one that is true is II. The others can easily be seen as false by examples. For example, let $f(x) = 1$ and $g(x) = 1$ with $a = 0$ and $b = 2$. Then I gives $2 = 4$ and III gives $2 = \sqrt{2}$, both false statements.
13. A period = $\frac{2\pi}{B} = \frac{2\pi}{3}$
14. A Let $u = x^3 + 1$. Then $\int \frac{3x^2}{\sqrt{x^3 + 1}} dx = \int u^{-1/2} du = 2u^{1/2} + C = 2\sqrt{x^3 + 1} + C$
15. D $f'(x) = (x-3)^2 + 2(x-2)(x-3) = (x-3)(3x-7)$; $f'(x)$ changes from positive to negative at $x = \frac{7}{3}$.
16. B $y' = 2 \frac{\sec x \tan x}{\sec x} = 2 \tan x$; $y'(\pi/4) = 2 \tan(\pi/4) = 2$. The slope of the normal line $-\frac{1}{y'(\pi/4)} = -\frac{1}{2}$
17. E Expand the integrand. $\int (x^2 + 1)^2 dx = \int (x^4 + 2x^2 + 1) dx = \frac{1}{5}x^5 + \frac{2}{3}x^3 + x + C$
18. D Want c so that $f'(c) = \frac{f\left(\frac{3\pi}{2}\right) - f\left(\frac{\pi}{2}\right)}{\frac{3\pi}{2} - \frac{\pi}{2}} = \frac{\sin\left(\frac{3\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)}{\pi} = \frac{0}{\pi}$.
 $f'(c) = \frac{1}{2} \cos\left(\frac{c}{2}\right) = 0 \Rightarrow c = \pi$
19. E The only one that is true is E. A consideration of the graph of $y = f(x)$, which is a standard cubic to the left of 0 and a line with slope 1 to the right of 0, shows the other options to be false.