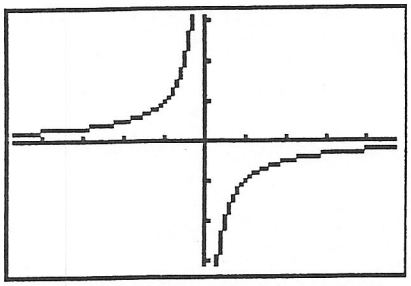
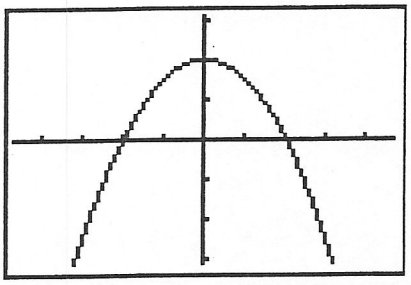
Graphs of f, f', f''.

Match the five functions a-e given below with their derivatives (i)-(v). Explain your reasoning.

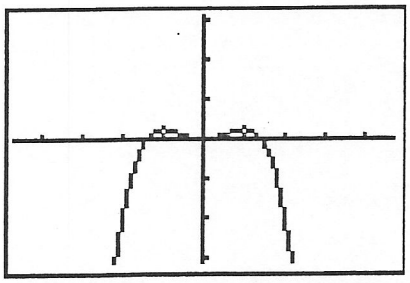
a.



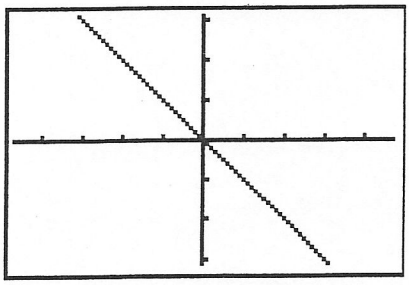
b.



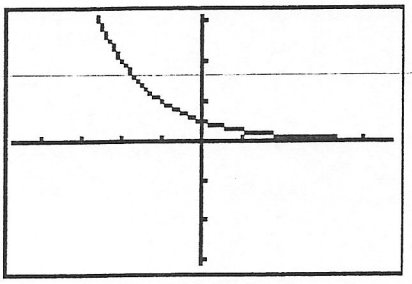
c.



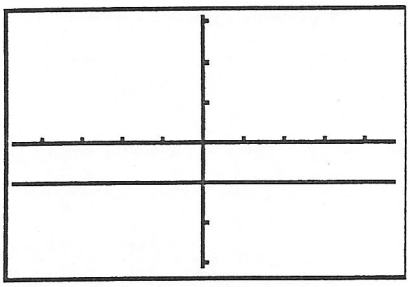
d.



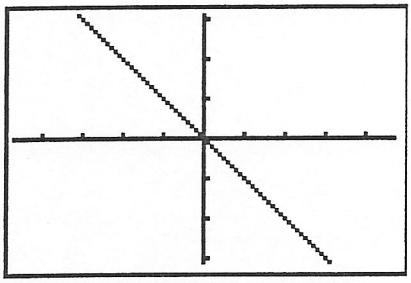
e.



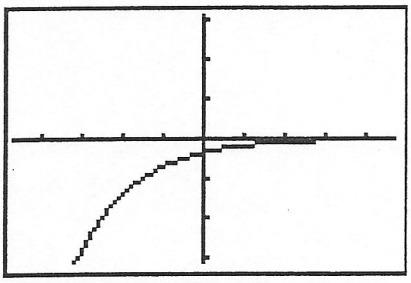
(i)



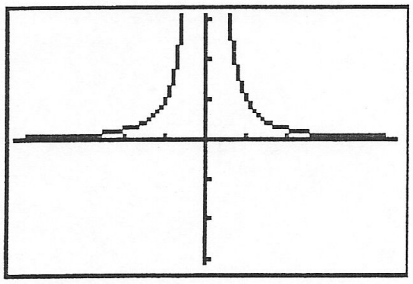
(ii)



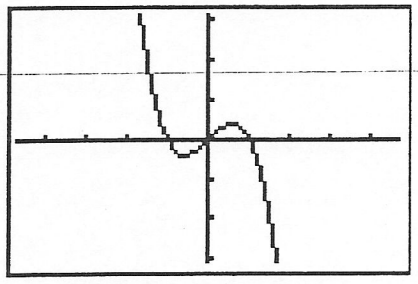
(iii)



(iv)



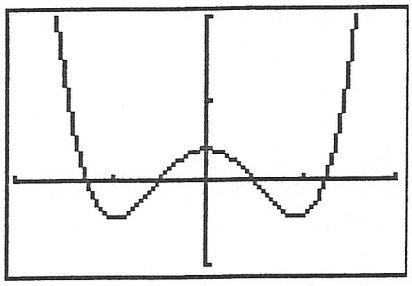
(v)



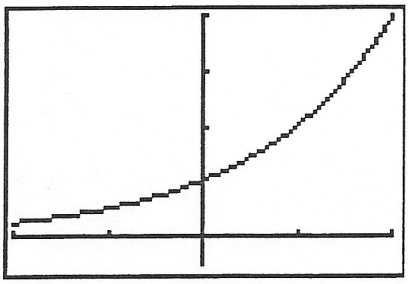
Graphs of f, f', f''.

e graphs of some functions are given below. Indicate on what intervals the functions are increasing and on what intervals they are decreasing, and then sketch the derivatives.

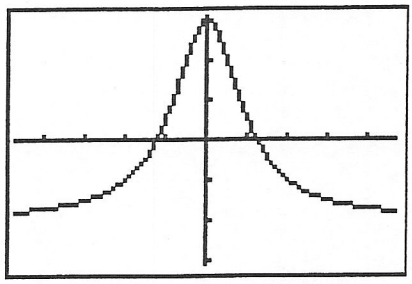
1.



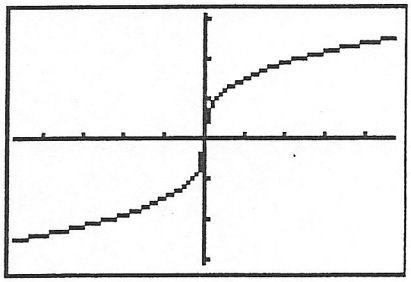
2.

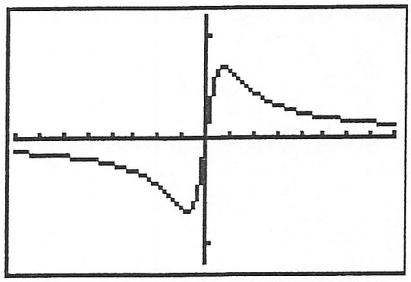


3.

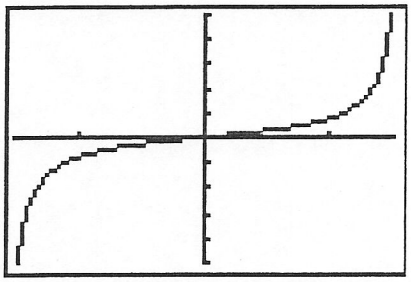


4.

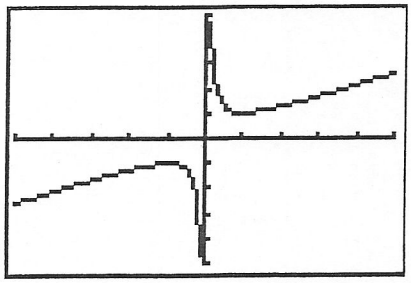




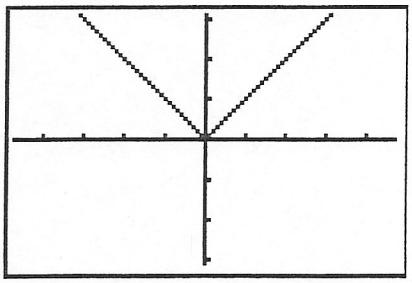
6.



7.



8.



Repeat the exercise, except this time the graphs are of $y = f'(x)$. Indicate on what intervals the functions e increasing and on what intervals they are decreasing, and then sketch a graph of the function.

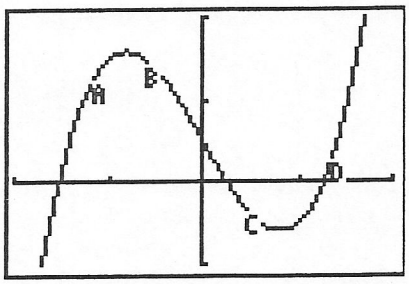
Graphs of f, f', f''.

A function f is continuous on the interval $[-3, 3]$ and its first and second derivatives have the values given in the following table:

x	$(-3,-1)$	-1	$(-1,0)$	0	$(0,1)$	1	$(1,3)$
$f'(x)$	positive	0	negative	negative	negative	0	negative
$f''(x)$	negative	negative	negative	0	positive	0	negative

a. What are the x-coordinates of all the relative maxima and minima of f on $[-3, 3]$? Justify your answer.
b. What are the x-coordinates of all points of inflection of f on the interval $[-3, 3]$? Justify your answer.
c. Sketch a possible graph for f which satisfies all of the given properties.

2. The graph of a function f together with some points on the graph is given below.



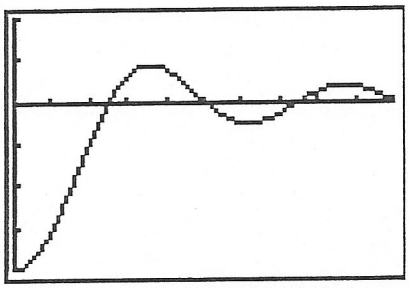
a. At which point(s) is the first derivative of f positive?
b. At which point(s) is the second derivative of f positive?

3. Repeat problem 2, but this time the graph is of a function f'.

4. For each part, determine whether it is possible for a twice differentiable function f to satisfy al of the properties listed in that part. Justify your answers.

a. $f'(x) > 0$, $f''(x) > 0$ but $f(x) < 0$ for all real numbers x
b. $f''(x) > 0$ but $f'(x) < 0$ for all real numbers x
c. $f''(x) > 0$ but $f(x) < 0$ for all real numbers x

5. The graph below is the graph of the derivative of a function f. Use this graph to answer the following questions about f on the interval $(0, 10)$. In each case, be sure to justify your answer.



a. On what subinterval(s) is f increasing?
b. On what subinterval(s) is f decreasing?
c. Find the x-coordinates of all relative minima of f.
d. Find the x-coordinates of all relative maxima of f.
e. On what subinterval(s) is f concave up?
f. On what subinterval(s) is f concave down?
g. Find the x-coordinates of all points of inflection of f.

6. Let f be a function which is twice differentiable for all real numbers and which satisfies the properties below. In addition, let $g(x) = f(x^2)$.

(i) $f(0) = 1$
(ii) $f'(x) > 0$ for all $x \neq 0$
(iii) f is concave down for all $x < 0$ and is concave up for all $x > 0$.

a. Sketch a possible graph for f which takes into account its properties given above.
b. Find the x-coordinates of all relative minimum point(s) of g. Justify your answer.
c. Where is the graph of g concave up? Justify your answer.
d. Use the information obtained in the three previous parts to sketch a possible graph of g.