## AP® CALCULUS AB 2014 SCORING GUIDELINES

## Question 5

x	-2	-2 < x < -1	-1	-1 < x < 1	1	1 < x < 3	3
f(x)	12	Positive	8	Positive	2	Positive	7
f'(x)	-5	Negative	0	Negative	0	Positive	1/2
g(x)	-1	Negative	0	Positive	3	Positive	1
g'(x)	2	Positive	3 2	Positive	0	Negative	-2

The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table above.

(a) Find the x-coordinate of each relative minimum of f on the interval [-2, 3]. Justify your answers.

(b) Explain why there must be a value c, for -1 < c < 1, such that f''(c) = 0.

(c) The function h is defined by  $h(x) = \ln(f(x))$ . Find h'(3). Show the computations that lead to your answer.

(d) Evaluate  $\int_{-2}^{3} f'(g(x))g'(x) dx$ .

(a) x = 1 is the only critical point at which f' changes sign from negative to positive. Therefore, f has a relative minimum at x = 1.

1: answer with justification

(b) f' is differentiable  $\Rightarrow f'$  is continuous on the interval  $-1 \le x \le 1$   $\frac{f'(1) - f'(-1)}{1 - (-1)} = \frac{0 - 0}{2} = 0$ 

 $2 = \begin{cases} 1 : f'(1) - f'(-1) = 0 \\ 1 : \text{ explanation, using Mean Value Theorem} \end{cases}$ 

Therefore, by the Mean Value Theorem, there is at least one value c, -1 < c < 1, such that f''(c) = 0.

(c)  $h'(x) = \frac{1}{f(x)} \cdot f'(x)$  $h'(3) = \frac{1}{f(3)} \cdot f'(3) = \frac{1}{7} \cdot \frac{1}{2} = \frac{1}{14}$ 

 $3: \begin{cases} 2: h'(x) \\ 1: \text{answer} \end{cases}$ 

(d)  $\int_{-2}^{3} f'(g(x))g'(x) dx = \left[ f(g(x)) \right]_{x=-2}^{x=3}$ = f(g(3)) - f(g(-2))= f(1) - f(-1)= 2 - 8 = -6

 $3: \left\{ \begin{array}{l} 2: Fundamental \ Theorem \ of \ Calculus \\ 1: answer \end{array} \right.$