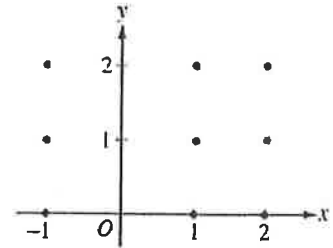


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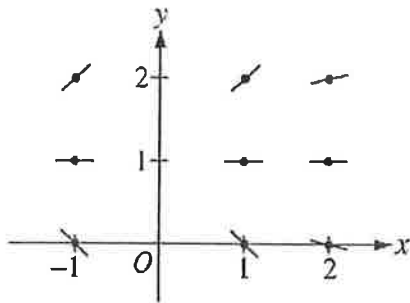
Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)
- (b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.
- (c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \rightarrow \infty} f(x)$.



(a)



2 : $\begin{cases} 1 : \text{zero slopes} \\ 1 : \text{all other slopes} \end{cases}$

(b) $\frac{1}{y-1} dy = \frac{1}{x^2} dx$

$$\ln|y-1| = -\frac{1}{x} + C$$

$$|y-1| = e^{-\frac{1}{x} + C}$$

$$|y-1| = e^C e^{-\frac{1}{x}}$$

$$y-1 = ke^{-\frac{1}{x}}, \text{ where } k = \pm e^C$$

$$-1 = ke^{-\frac{1}{2}}$$

$$k = -e^{\frac{1}{2}}$$

$$f(x) = 1 - e^{\left(\frac{1}{2} - \frac{1}{x}\right)}, x > 0$$

6 : $\begin{cases} 1 : \text{separates variables} \\ 2 : \text{antidifferentiates} \\ 1 : \text{includes constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

(c) $\lim_{x \rightarrow \infty} 1 - e^{\left(\frac{1}{2} - \frac{1}{x}\right)} = 1 - \sqrt{e}$

1 : limit