AP® CALCULUS AB
2016 SCORING GUIDELINES

Question 4

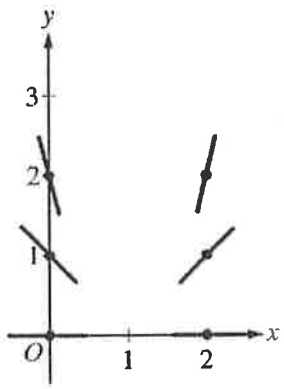
Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.

(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

(b) $\left.\frac{dy}{dx}\right|_{(x,\,y)=(2,\,3)} = \frac{3^2}{2-1} = 9$

An equation for the tangent line is $y = 9(x-2) + 3$.

$f(2.1) \approx 9(2.1-2) + 3 = 3.9$

$2 : \begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

(c) $\frac{1}{y^2}\,dy = \frac{1}{x-1}\,dx$

$\int \frac{1}{y^2}\,dy = \int \frac{1}{x-1}\,dx$

$-\frac{1}{y} = \ln|x-1| + C$

$-\frac{1}{3} = \ln|2-1| + C \Rightarrow C = -\frac{1}{3}$

$-\frac{1}{y} = \ln|x-1| - \frac{1}{3}$

$y = \frac{1}{\frac{1}{3} - \ln(x-1)}$

Note: This solution is valid for $1 < x < 1 + e^{1/3}$.

$5 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/5 [1-2-0-0] if no constant of integration

Note: 0/5 if no separation of variables