

AP[®] CALCULUS AB
2009 SCORING GUIDELINES

Question 3

Mighty Cable Company manufactures cable that sells for \$120 per meter. For a cable of fixed length, the cost of producing a portion of the cable varies with its distance from the beginning of the cable. Mighty reports that the cost to produce a portion of a cable that is x meters from the beginning of the cable is $6\sqrt{x}$ dollars per meter. (Note: Profit is defined to be the difference between the amount of money received by the company for selling the cable and the company's cost of producing the cable.)

- (a) Find Mighty's profit on the sale of a 25-meter cable.
- (b) Using correct units, explain the meaning of $\int_{25}^{30} 6\sqrt{x} \, dx$ in the context of this problem.
- (c) Write an expression, involving an integral, that represents Mighty's profit on the sale of a cable that is k meters long.
- (d) Find the maximum profit that Mighty could earn on the sale of one cable. Justify your answer.

(a) Profit = $120 \cdot 25 - \int_0^{25} 6\sqrt{x} \, dx = 2500$ dollars

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $\int_{25}^{30} 6\sqrt{x} \, dx$ is the difference in cost, in dollars, of producing a cable of length 30 meters and a cable of length 25 meters.

1 : answer with units

(c) Profit = $120k - \int_0^k 6\sqrt{x} \, dx$ dollars

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{expression} \end{cases}$

(d) Let $P(k)$ be the profit for a cable of length k .
 $P'(k) = 120 - 6\sqrt{k} = 0$ when $k = 400$.
 This is the only critical point for P , and P' changes from positive to negative at $k = 400$.
 Therefore, the maximum profit is $P(400) = 16,000$ dollars.

4 : $\begin{cases} 1 : P'(k) = 0 \\ 1 : k = 400 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$