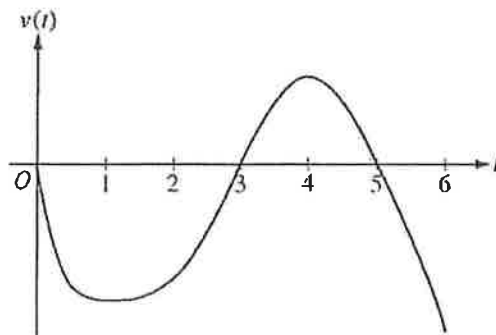


**AP[®] CALCULUS AB
2008 SCORING GUIDELINES**

Question 4



Graph of v

A particle moves along the x -axis so that its velocity at time t , for $0 \leq t \leq 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t -axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

- (a) For $0 \leq t \leq 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.
- (b) For how many values of t , where $0 \leq t \leq 6$, is the particle at $x = -8$? Explain your reasoning.
- (c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.
- (d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

- (a) Since $v(t) < 0$ for $0 < t < 3$ and $5 < t < 6$, and $v(t) > 0$ for $3 < t < 5$, we consider $t = 3$ and $t = 6$.

$$x(3) = -2 + \int_0^3 v(t) dt = -2 - 8 = -10$$

$$x(6) = -2 + \int_0^6 v(t) dt = -2 - 8 + 3 - 2 = -9$$

Therefore, the particle is farthest left at time $t = 3$ when its position is $x(3) = -10$.

- (b) The particle moves continuously and monotonically from $x(0) = -2$ to $x(3) = -10$. Similarly, the particle moves continuously and monotonically from $x(3) = -10$ to $x(5) = -7$ and also from $x(5) = -7$ to $x(6) = -9$.

By the Intermediate Value Theorem, there are three values of t for which the particle is at $x(t) = -8$.

- (c) The speed is decreasing on the interval $2 < t < 3$ since on this interval $v < 0$ and v is increasing.
- (d) The acceleration is negative on the intervals $0 < t < 1$ and $4 < t < 6$ since velocity is decreasing on these intervals.

$$3 : \begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{considers } \int_0^6 v(t) dt \\ 1 : \text{conclusion} \end{cases}$$

$$3 : \begin{cases} 1 : \text{positions at } t = 3, t = 5, \\ \quad \text{and } t = 6 \\ 1 : \text{description of motion} \\ 1 : \text{conclusion} \end{cases}$$

1 : answer with reason

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$