Question 2

For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\dfrac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

(a) At time $t = 4$, is the particle speeding up or slowing down?

(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the position of the particle at time $t = 0$.

(d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

(a) $v(4) = 2.978716 > 0$
$v'(4) = -1.164000 < 0$

The particle is slowing down since the velocity and acceleration have different signs.

2 : conclusion with reason

(b) $v(t) = 0 \Rightarrow t = 2.707468$

$v(t)$ changes from positive to negative at $t = 2.707$. Therefore, the particle changes direction at this time.

$2 : \begin{cases} 1 : t = 2.707 \\ 1 : \text{justification} \end{cases}$

(c) $x(0) = x(4) + \displaystyle\int_4^0 v(t)\, dt$
$= 2 + (-5.815027) = -3.815$

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

(d) Distance $= \displaystyle\int_0^3 |v(t)|\, dt = 5.301$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$