AP® CALCULUS AB
2012 SCORING GUIDELINES

Question 4

The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\,dx$.

(a) $f'(x) = \frac{1}{2}\left(25 - x^2\right)^{-1/2}(-2x) = \frac{-x}{\sqrt{25 - x^2}}, \quad -5 < x < 5$

2 : $f'(x)$

(b) $f'(-3) = \frac{3}{\sqrt{25 - 9}} = \frac{3}{4}$

$f(-3) = \sqrt{25 - 9} = 4$

An equation for the tangent line is $y = 4 + \frac{3}{4}(x + 3)$.

2 : $\begin{cases} 1 : f'(-3) \\ 1 : \text{answer} \end{cases}$

(c) $\lim\limits_{x \to -3^-} g(x) = \lim\limits_{x \to -3^-} f(x) = \lim\limits_{x \to -3^-} \sqrt{25 - x^2} = 4$

$\lim\limits_{x \to -3^+} g(x) = \lim\limits_{x \to -3^+} (x + 7) = 4$

Therefore, $\lim\limits_{x \to -3} g(x) = 4$.

$g(-3) = f(-3) = 4$

So, $\lim\limits_{x \to -3} g(x) = g(-3)$.

Therefore, g is continuous at $x = -3$.

2 : $\begin{cases} 1 : \text{considers one-sided limits} \\ 1 : \text{answer with explanation} \end{cases}$

(d) Let $u = 25 - x^2 \Rightarrow du = -2x\,dx$

$$\int_0^5 x\sqrt{25 - x^2}\,dx = -\frac{1}{2}\int_{25}^{0} \sqrt{u}\,du$$

$$= \left[-\frac{1}{2} \cdot \frac{2}{3} u^{3/2}\right]_{u=25}^{u=0}$$

$$= -\frac{1}{3}(0 - 125) = \frac{125}{3}$$

3 : $\begin{cases} 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

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