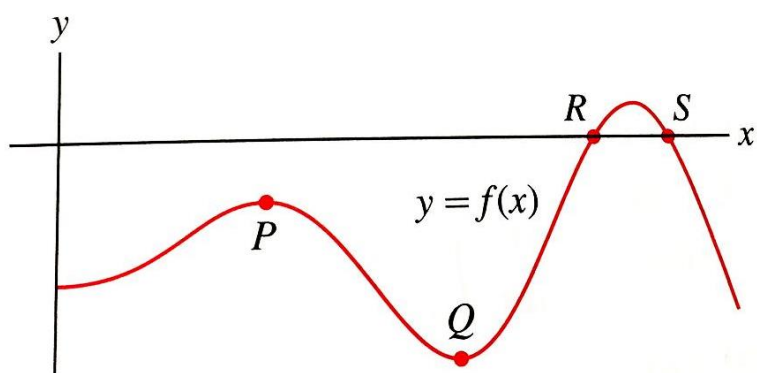



39. Let  $A(x) = \int_0^x f(t) dt$ , with  $f(x)$  as in Figure 11.

- Does  $A(x)$  have a local maximum at  $P$ ?
- Where does  $A(x)$  have a local minimum?
- Where does  $A(x)$  have a local maximum?
- True or false?  $A(x) < 0$  for all  $x$  in the interval shown.



**FIGURE 11** Graph of  $f(x)$ .

In Exercises 43–44, let  $A(x) = \int_a^x f(t) dt$ .

43.  **Area Functions and Concavity** Explain why the following statements are true. Assume  $f(x)$  is differentiable.

- If  $c$  is an inflection point of  $A(x)$ , then  $f'(c) = 0$ .
- $A(x)$  is concave up if  $f(x)$  is increasing.
- $A(x)$  is concave down if  $f(x)$  is decreasing.

45. Let  $A(x) = \int_0^x f(t) dt$ , with  $f(x)$  as in Figure 12. Determine:

- (a) The intervals on which  $A(x)$  is increasing and decreasing
- (b) The values  $x$  where  $A(x)$  has a local min or max
- (c) The inflection points of  $A(x)$
- (d) The intervals where  $A(x)$  is concave up or concave down

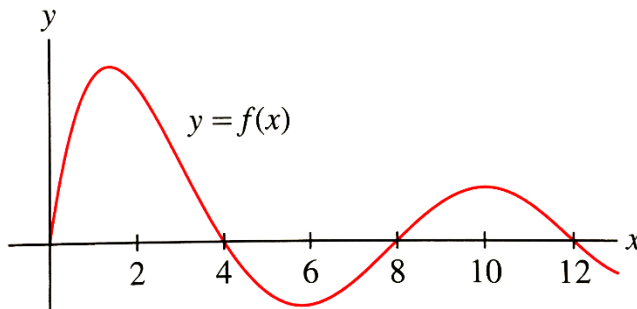


FIGURE 12

3. A survey shows that a mayoral candidate is gaining votes at a rate of  $2000t + 1000$  votes per day, where  $t$  is the number of days since she announced her candidacy. How many supporters will the candidate have after 60 days, assuming that she had no supporters at  $t = 0$ ?

5. Find the displacement of a particle moving in a straight line with velocity  $v(t) = 4t - 3$  m/s over the time interval  $[2, 5]$ .

Solutions

- 39.** (a)  $A(x)$  does not have a local maximum at  $P$ .  
(b)  $A(x)$  has a local minimum at  $R$ .  
(c)  $A(x)$  has a local maximum at  $S$ .  
(d) True.

**43.** (a) If  $x = c$  is an inflection point of  $A(x)$ , then  
 $A''(c) = f'(c) = 0$ .

(b) If  $A(x)$  is concave up, then  $A''(x) > 0$ . Since  $A(x)$  is the area function associated with  $f(x)$ ,  $A'(x) = f(x)$  by FTC II, so  $A''(x) = f'(x)$ . Therefore  $f'(x) > 0$ , so  $f(x)$  is increasing.

(c) If  $A(x)$  is concave down, then  $A''(x) < 0$ . Since  $A(x)$  is the area function associated with  $f(x)$ ,  $A'(x) = f(x)$  by FTC II, so  $A''(x) = f'(x)$ . Therefore,  $f'(x) < 0$  and so  $f(x)$  is decreasing.

**45.** (a)  $A(x)$  is increasing on the intervals  $(0, 4)$  and  $(8, 12)$  and is decreasing on the intervals  $(4, 8)$  and  $(12, \infty)$ .

(b) Local minimum:  $x = 8$ ; local maximum:  $x = 4$  and  $x = 12$ .

(c)  $A(x)$  has inflection points at  $x = 2$ ,  $x = 6$ , and  $x = 10$ .

(d)  $A(x)$  is concave up on the intervals  $(0, 2)$  and  $(6, 10)$  and is concave down on the intervals  $(2, 6)$  and  $(10, \infty)$ .

- 3.** 3,660,000    **5.** 33 meters