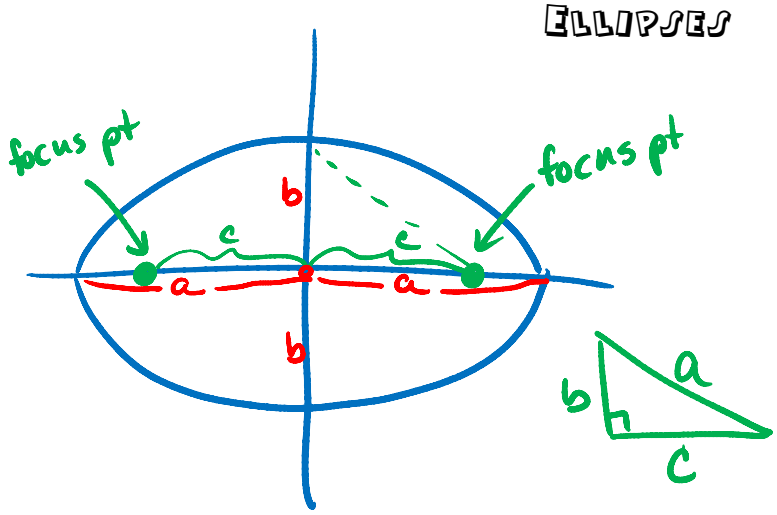
Conic Sections: Day 2

Ellipses



center (h,k)

$2a$ = major axis

$2b$ = minor axis

c = focal length

$c^2 = a^2 - b^2$

Equations of an Ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

If major axis is horizontal

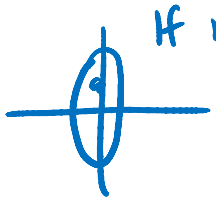
Parametrics

$X = a\cos T + h$

$Y = b\sin T + k$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

If major axis is vertical



Parametrics

$X = b\cos T + h$

$Y = a\sin T + k$

$a > b$

1. $\frac{x^2}{9} + \frac{y^2}{16} = 1$

b^2 $\quad$ a^2 vertical

$a = 4$

$b = 3$

center $(0,0)$

length of major axis = 8

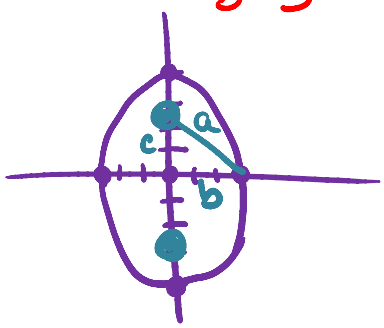
length of minor axis = 6

Coords

$(0, \pm 4)$

$(\pm 3, 0)$

foci $\longrightarrow$ $(0, \pm\sqrt{7})$



$c^2 = a^2 - b^2$

$c^2 = 16 - 9$

$c = \sqrt{7}$

2. $x^2 + 25y^2 = 100$

$\frac{x^2}{100} + \frac{25y^2}{100} = \frac{100}{100}$ Need = 1

$\frac{x^2}{100} + \frac{y^2}{4} = 1$

a^2 b^2

horizontal

$2a = 20$

$2b = 4$

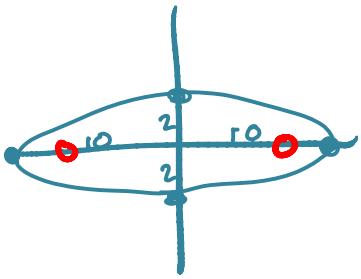
$c = \sqrt{96}$

Coords of

major axis: $(\pm 10, 0)$

minor axis: $(0, \pm 2)$

foci: $(\pm\sqrt{96}, 0)$



3. $\frac{(x+1)^2}{16} + \frac{(y-4)^2}{36} = 1$

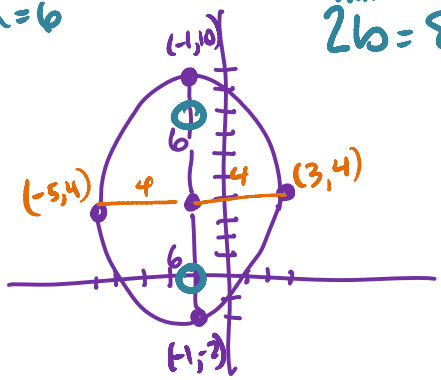
b^2 a^2 vertical

$b = 4$ $a = 6$

center $(-1, 4)$

major $2a = 12$

minor $2b = 8$



Coords

major axis $(-1, 4 \pm 6) =$

$(-1, 10)(-1, -2)$

minor axis $(-1 \pm 4, 4) =$

$(3, 4)(-5, 4)$

foci

$c^2 = a^2 - b^2$

$c^2 = 36 - 16$

$c = \sqrt{20}$

Coords:

$(-1, 4 \pm \sqrt{20})$

4. Parametric Form: $X = 2 + 4\cos T$

$Y = 5 + 3\sin T$

$a = 4$ (hor.)

$b = 3$

center $(2, 5)$

$\frac{(x-2)^2}{16} + \frac{(y-5)^2}{9} = 1$