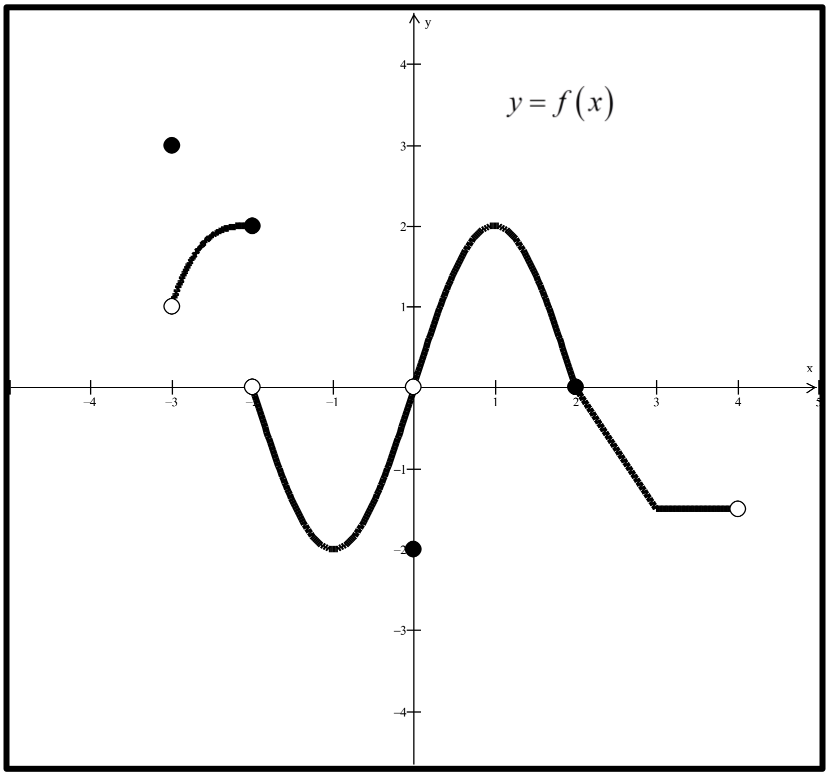
Cumulative Review
Quarter 1

Name Key

Part 1: Chapter 2



1. Use the diagram above to determine the following.

a. $\lim_{x\to 2^-} f(x) =$ 0

b. $\lim_{x\to 2} f(x) =$ 0

c. $\lim_{x\to -3} f(x) =$ 1

d. $f(-3) =$ 3

e. $\lim_{x\to -2} f(x) =$ DNE

f. $f(0) =$ -2

2. Find the points of discontinuity of $f(x)=\dfrac{9x^2-16}{3x^2-x-4}$. For each discontinuity, identify the type.

$$f(x)=\frac{(3x+4)(3x-4)}{(3x-4)(x+1)}=\frac{3x+4}{x+1},\quad x\neq \tfrac{4}{3},-1$$

$\boxed{\text{removable @ } x=4/3 \qquad \text{infinite @ } x=-1}$

3. Find the limit of the following;

a. $\lim\limits_{x\to 0}\left(\dfrac{2x+\sin 3x}{x}\right)$

$$=\lim_{x\to 0}\frac{2x}{x}+\lim_{x\to 0}\frac{3\sin 3x}{3x}$$
$$=\lim_{x\to 0}2+3\lim_{x\to 0}\frac{\sin 3x}{3x}$$
$$=2+3\cdot 1=\boxed{5}$$

b. $\lim\limits_{x\to \frac{4}{3}}\left(\dfrac{9x^2-16}{3x^2-x-4}\right)$

$$=\lim_{x\to\frac{4}{3}}\left(\frac{3x+4}{x+1}\right)$$
$$=\frac{3\left(\frac{4}{3}\right)+4}{\frac{4}{3}+1}=\frac{8}{\frac{7}{3}}=8\cdot\frac{3}{7}=\boxed{\frac{24}{7}}$$

c. $\lim\limits_{x\to \infty}\left(\dfrac{9x^2-16}{3x^2-x-4}\right)$

Use E.B.M. $y=\dfrac{9x^2}{3x^2}=3$

$$=\boxed{3}$$

d. $\lim\limits_{x\to -\infty}\left(\dfrac{3-4x^5}{x^3-1}\right)$

Use E.B.M $y=\dfrac{-4x^5}{x^3}=-4x^2$

$$=\boxed{-\infty}$$

e. $\lim\limits_{x\to 0}\left(\dfrac{-2\tan x}{x}\right)=\lim\limits_{x\to 0}\dfrac{-2\sin x}{x\cos x}$

$$=\lim_{x\to 0}\frac{-2}{\cos x}\cdot\lim_{x\to 0}\frac{\sin x}{x}$$
$$=\frac{-2}{1}\cdot 1=\boxed{-2}$$

4. Extend the function $f(x)=\dfrac{x^2-4x-32}{x+4}$ to make it continuous.

$$f(x)=\frac{(x-8)(x+4)}{(x+4)}$$

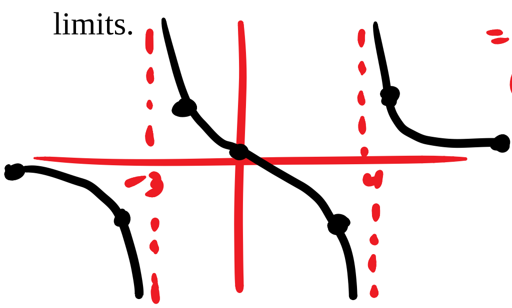
$\boxed{g(x)=x-8}$

OR

$$f(x)=\begin{cases}\dfrac{x^2-4x-32}{x+4}, & x\neq -4\\ -12, & x=4\end{cases}$$

5. Describe the behavior of $f(x)=\dfrac{3x-1}{x^2-x-12}$ to the right and left of the vertical asymptote(s) using limits.

$$=\frac{3x-1}{(x-4)(x+3)}$$



x	y
-10	-31/98
-4	-13/8
-2	7/6
0	1/12

x	y
3	-4/3
5	7/4
10	29/78

$$\lim_{x\to -3^-} f(x)=-\infty$$
$$\lim_{x\to -3^+} f(x)=+\infty$$
$$\lim_{x\to 4^-} f(x)=-\infty$$
$$\lim_{x\to 4^+} f(x)=+\infty$$

6. Write the equation of the tangent line to $f(x) = -3x^2 - x + 1$ at x= -2. (Use a limit definition to do this.)

$$f'(x) = \lim_{h \to 0} \frac{[-3(x+h)^2 - (x+h) + 1] - [-3x^2 - x + 1]}{h}$$

$$= \lim_{h \to 0} \frac{-6xh - 3h^2 - h}{h}$$

$$= \lim_{h \to 0} -6x - 3h - 1$$

$$= -6x - 1$$

$$f'(-2) = -6(-2) - 1 = 11$$

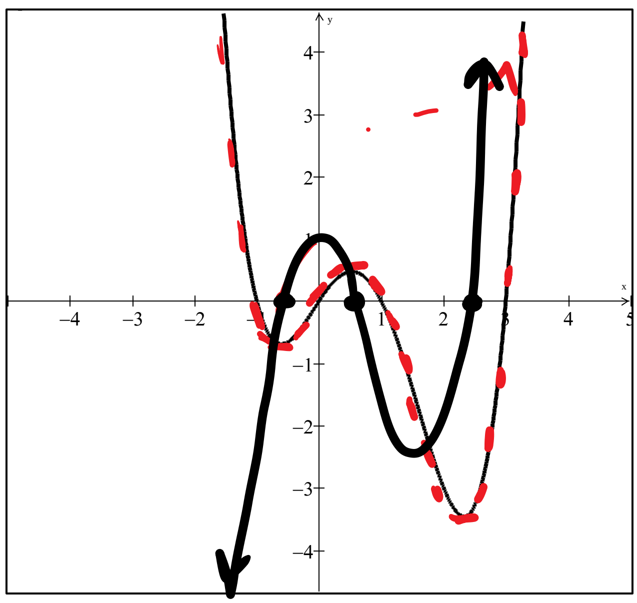
$$f(-2) = -3(-2)^2 - (-2) + 1 = -9$$

$$\boxed{y + 9 = 11(x+2) \quad \text{OR} \quad y = 11x + 13}$$

Part 2: Chapter 3.1-3.5

★ 1. Solution for #1 is @ end of this key.

2. Sketch a graph of f'(x) if the function, f(x), is shown below.



3. Find dy/dx of each of the following:

a) $y = \frac{x^2 + 5x - 1}{x^3}$

$$\frac{dy}{dx} = \frac{x^3(2x+5) - (x^2 + 5x - 1)(3x^2)}{(x^3)^2}$$

$$= \frac{2x^4 + 5x^3 - 3x^4 - 15x^3 + 3x^2}{x^6}$$

$$= \frac{-x^4 - 10x^3 + 3x^2}{x^6} = \boxed{\frac{-x^2 - 10x + 3}{x^4}}$$

b) $y = 4x^3 - 7x + 6\sqrt{x} - \frac{4}{x^3}$

$$\frac{dy}{dx} = 12x^2 - 7 + 3x^{-1/2} + \frac{12}{x^4}$$

$$= \boxed{12x^2 - 7 + \frac{3}{\sqrt{x}} + \frac{12}{x^4}}$$

c) $y = \sin x \cdot \cos x$ product rule

$$\frac{dy}{dx} = \sin x(-\sin x) + \cos x(\cos x)$$

$$= \boxed{-\sin^2 x + \cos^2 x}$$

d) $y = \frac{\cot x}{x^3}$ quotient rule

$$\frac{dy}{dx} = \frac{x^3 \cdot (-\csc^2 x) - \cot x (3x^2)}{(x^3)^2}$$

$$= \boxed{\frac{-x^3 \csc^2 x - 3x^2 \cot x}{x^6}}$$

e) $y = \dfrac{\sin x}{1+\tan x}$

$$\frac{dy}{dx} = \frac{(1+\tan x)\cos x - \sin x\left(\sec^2 x\right)}{(1+\tan x)^2}$$

$$= \boxed{\frac{\cos x + \sin x - \tan x \sec x}{(1+\tan x)^2}}$$

f) $y = (1+\cos x)(1-\cos x)$

$y = 1 - \cos^2 x$

$y = \sin^2 x$

$y = \sin x \cdot \sin x$

$$\frac{dy}{dx} = \sin x(\cos x) + \sin x(\cos x)$$

$$= \boxed{2\sin x \cos x}$$

4. Find the equation of the tangent line and normal line to $y = 4\cos x$ at $x = \dfrac{2\pi}{3}$.

Tangent line

$y' = -4\sin x$

$y'\left(\frac{2\pi}{3}\right) = -4\sin\left(\frac{2\pi}{3}\right)$

$= -4\left(\frac{\sqrt{3}}{2}\right)$

$= -2\sqrt{3}$ ← slope

$y\left(\frac{2\pi}{3}\right) = 4\cos\left(\frac{2\pi}{3}\right)$

$= 4\left(-\frac{1}{2}\right) = -2$ ← pt $\left(\frac{2\pi}{3}, -2\right)$

Tangent line:

$$\boxed{y + 2 = -2\sqrt{3}\left(x - \frac{2\pi}{3}\right)}$$

Normal line:

$$\boxed{y + 2 = \frac{1}{2\sqrt{3}}\left(x - \frac{2\pi}{3}\right)}$$

5. Find $\dfrac{d^{101}y}{dx^{101}}$ of y = sinx.

Pattern:

$\dfrac{dy}{dx} = \cos x$

$\dfrac{d^2y}{dx^2} = -\sin x$

$\dfrac{d^3y}{dx^3} = -\cos x$

multiples of 4 → $\dfrac{d^4y}{dx^4} = \sin x$

$\dfrac{d^5y}{dx^5} = \cos x$

etc

$$\boxed{\frac{d^{101}y}{dx^{101}} = \cos x}$$

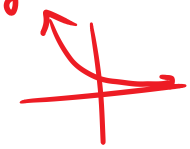
<u>Calculator OK</u>

Part 5: Chapter 2

1. Find the limit using the graph.

a. $\lim\limits_{x\to 0^+} 2^{-\frac{1}{x}} = \lim\limits_{x\to +\infty} 2^{-x}$

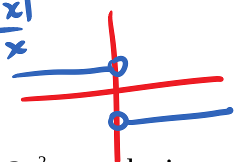
$y = 2^{-x}$



$= \boxed{0}$

b. $\lim\limits_{x\to 0^-} \frac{|-x|}{-x} = \boxed{1}$

$y = \frac{|-x|}{-x}$



2. Find the average rate of change of $f(x) = x - 3x^2$ over the interval $[-4,-1]$.

$$\text{av. rate of change} = \frac{f(-1) - f(-4)}{-1 - {-4}}$$

$$= \frac{-4 - (-52)}{3}$$

$$= \boxed{16}$$

Check: $f(x) = x - 3x^2$
$f'(x) = 1 - 6x$
$f'(-1) = 1 - 6(-1) = \boxed{7}$

3. For $f(x) = x - 3x^2$

a. Find the slope of the curve at x= -1. Show all your calculations using LIMITS.

ALT. DEF. OF DER. @ PT.

$$f'(-1) = \lim_{x \to -1} \frac{f(x) - f(-1)}{x - (-1)} = \lim_{x \to -1} \frac{(x - 3x^2) - (-1 - 3(-1)^2)}{x+1} = \lim_{x \to -1} \frac{-3x^2 + x + 4}{x+1} = \lim_{x \to -1} \frac{(-3x+4)\cancel{(x+1)}}{\cancel{x+1}} = -3(-1) + 4 = \boxed{7}$$

b. Write the equation of the tangent to the curve at x= -1.

$f'(-1) = 7$
$f(-1) = -1 - 3(-1)^2 = -4$

$\boxed{y + 4 = 7(x+1) \text{ or } y = 7x + 3}$

c. Write the equation of the normal to the curve at x= -1.

$\boxed{y + 4 = -\frac{1}{7}(x+1)}$

Part 3: Chapter 3.1-3.5

1. On Earth, if you shoot a rubber band 64 feet straight up into the air, the rubber band will be $s(t) = 64t - 16t^2$ feet above your hand at t seconds after firing.

a) Find v(t) and a(t).

$\boxed{v(t) = 64 - 32t \quad a(t) = -32}$

b) How long does it take the paper clip to reach its maximum height? (Use calculus methods.)

Let $v = 0$ $\quad 0 = 64 - 32t$
$t = 2$

$\boxed{2 \text{ secs}}$

c) With what velocity does it leave your hand?

Let $t = 0$ $\quad v(0) = 64 - 32(0) = 64$

$\boxed{64 \text{ ft/sec}}$

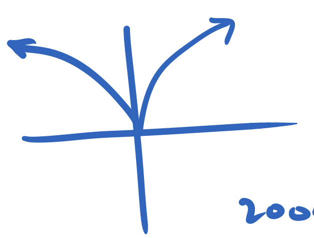
d) Is the paper clip speeding up, slowing down, or the same speed at t = 1 second? Justify.

$v(1) = 64 - 32(1) = 32$
$a(1) = -32$

$\boxed{\text{slowing down}}$

velocity and acceleration have opposite signs

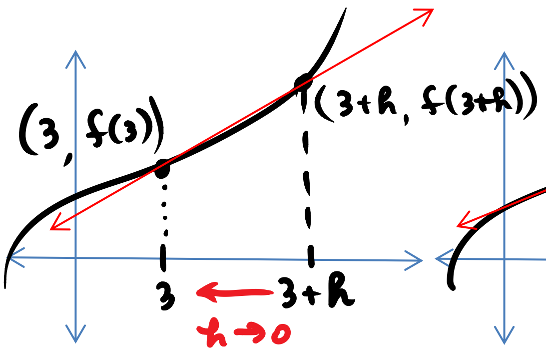
2. The function $f(x) = x^{\frac{4}{5}}$ is not differentiable at x = 0. What type of non-differentiability is it?



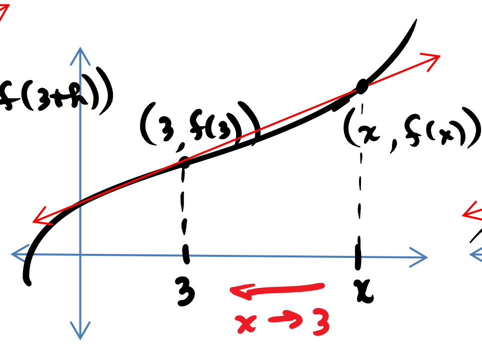
$\boxed{\text{cusp @ } x = 0}$

zoom in a few times

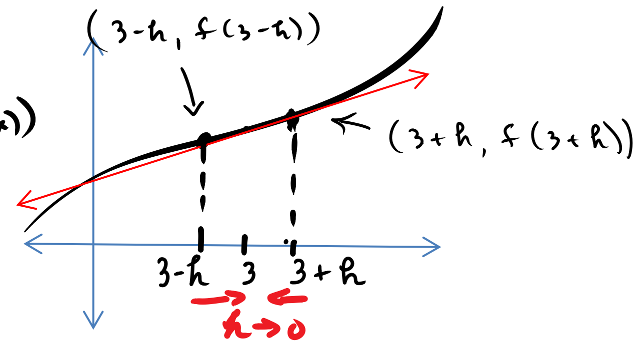
1. For each limit below, draw a labeled diagram that illustrates the geometric interpretation of each definition of derivative at x = 3. Write a sentence or two explaining each definition.



$$\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}$$



$$\lim_{x\to 3}\frac{f(x)-f(3)}{x-3}$$



$$\lim_{h\to 0}\frac{f(3+h)-f(3-h)}{2h}$$

The fraction in each problem is the slope of the secant line containing the two labeled points. Using the limit, the slopes of the secant lines will approach the slope of the tangent line, which is the derivative @ x = 3