Cumulative Review
Quarter 1

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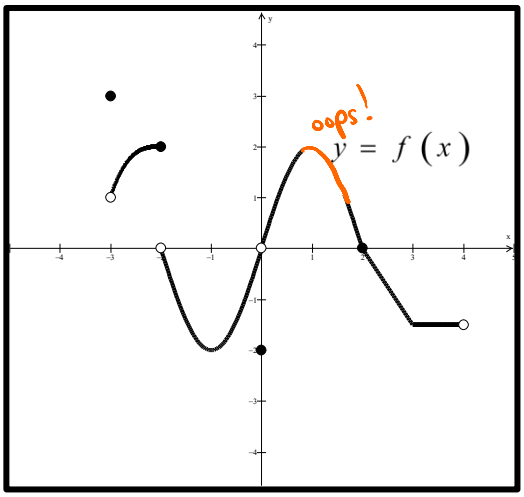
Part 1: Chapter 2



1. Use the diagram above to determine the following.

a. $\lim_{x\to 2^-} f(x) =$ 0

b. $\lim_{x\to 2} f(x) =$ 0

c. $\lim_{x\to -3} f(x) =$ 1

d. $f(-3) =$ 3

e. $\lim_{x\to -2} f(x) =$ DNE

f. $f(0) =$ −2

$\lim_{x\to 0} f(x) = 0$

2. Find the points of discontinuity of $f(x) = \dfrac{9x^2-16}{3x^2-x-4}$. For each discontinuity , identify the type.

$f(x) = \dfrac{(3x+4)(3x-4)}{(3x-4)(x+1)}$

Removable @ $\left(\frac{4}{3}, \frac{24}{7}\right)$

Infinite Disc. @ $x = -1$

$f\left(\frac{4}{3}\right) = \dfrac{3\left(\frac{4}{3}\right)+4}{\frac{4}{3}+1} = \dfrac{8}{\frac{7}{3}} = \dfrac{24}{7}$

3. Find the limit of the following;

a. $\lim_{x\to 0}\left(\dfrac{2x+\sin 3x}{x}\right) = \lim_{x\to 0}\left[\dfrac{2x}{x} + \dfrac{\sin 3x}{x}\right]$

$= \lim_{x\to 0}\left[2 + 3\dfrac{\sin 3x}{3x}\right]$

$= 2 + 3(1) = 5$

b. $\lim_{x\to \frac{4}{3}}\left(\dfrac{9x^2-16}{3x^2-x-4}\right) = \lim_{x\to \frac{4}{3}} \dfrac{(3x+4)(3x-4)}{(3x-4)(x+1)}$

$= \lim_{x\to \frac{4}{3}} \dfrac{3x+4}{x+1} = \dfrac{3\left(\frac{4}{3}\right)+4}{\frac{4}{3}+1} = \dfrac{24}{7}$

c. $\lim_{x\to\infty}\left(\dfrac{9x^2-16}{3x^2-x-4}\right)$ EBM $y = \dfrac{9x^2}{3x^2} = 3$

d. $\lim_{x\to-\infty}\left(\dfrac{3-4x^5}{x^3-1}\right)$ EBM $y = \dfrac{-4x^5}{x^3} = -4x^2$

$= -\infty$

e. $\lim_{x\to 0}\left(\dfrac{-2\tan x}{x}\right) = \lim_{x\to 0}\left[\dfrac{-2\sin x}{x\cos x}\right] = \lim_{x\to 0}\left[\dfrac{-2}{\cos x}\cdot\dfrac{\sin x}{x}\right]$

$= \dfrac{-2}{\cos 0}\cdot 1 = -2$

4. Extend the function $f(x)=\dfrac{x^2-4x-32}{x+4}$ to make it continuous.

$f(x)=\dfrac{(x+4)(x-8)}{x+4}=x-8$

$f(x)=\begin{cases}\dfrac{x^2-4x-32}{x+4}, & x\neq -4\\ x-8, & x=-4\end{cases}$

5. Describe the behavior of $f(x)=\dfrac{3x-1}{x^2-x-12}$ to the right and left of the vertical asymptote(s) using limits.

$f(x)=\dfrac{3x-1}{(x+3)(x-4)}$

V.A. @ $x=-3, 4$

$x=-3$

-3.1 $\lim\limits_{x\to -3^-} f(x)=-\infty$

-2.9 $\lim\limits_{x\to -3^+} f(x)=\infty$

$x=4$

3.9 $\lim\limits_{x\to 4^-} f(x)=-\infty$

4.1 $\lim\limits_{x\to 4^+} f(x)=\infty$

6. Write the equation of the tangent line to $f(x)=-3x^2-x+1$ at x= -2. (Use a limit definition to do this.)

Point $f(-2)=-3(-2)^2-(-2)+1=-9$

$(-2,-9)$

Slope: $\lim\limits_{h\to 0}\dfrac{-3(-2+h)^2-(-2+h)+1-(-9)}{h}$

$=\lim\limits_{h\to 0}\dfrac{-12+12h-3h^2+2-h+1+9}{h}$

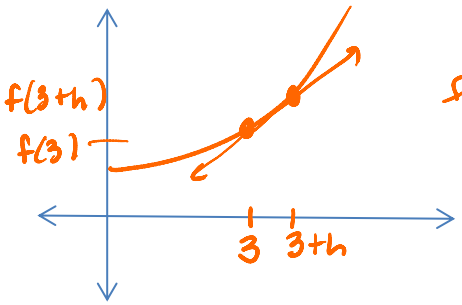
$=\lim\limits_{h\to 0}\dfrac{11h-3h^2}{h}$

$=\lim\limits_{h\to 0} 11-3h=11=$ slope

$y+9=11(x+2)$

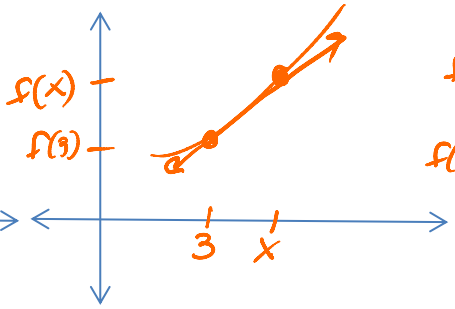
Part 2: Chapter 3.1-3.5

1. For each limit below, draw a labeled diagram that illustrates the geometric interpretation of each definition of derivative at x = 3. Write a sentence or two explaining each definition.



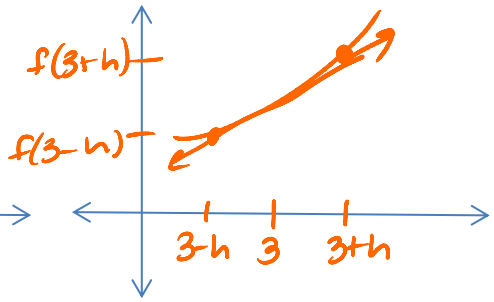
$\lim\limits_{h\to 0}\dfrac{f(3+h)-f(3)}{h}$

This is the slope between 2 points $(3, f(3))$ and $(3+h, f(3+h))$ as $h\to 0$.



$\lim\limits_{x\to 3}\dfrac{f(x)-f(3)}{x-3}$

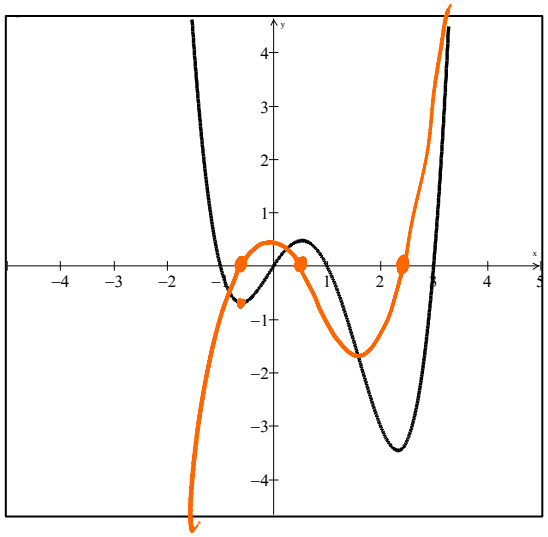
This is the slope between 2 pts $(3, f(3))$ and $(x, f(x))$ as $x\to 3$



$\lim\limits_{h\to 0}\dfrac{f(3+h)-f(3-h)}{2h}$

This is the slope between 2 points $(3-h, f(3-h))$ and $(3+h, f(3+h))$ as $h\to 0$.

2. Sketch a graph of f'(x) if the function, f(x), is shown below.



3. Find dy/dx of each of the following:

a) $y=\dfrac{x^2+5x-1}{x^3}$ Quotient Rule or Divide 1st

$\dfrac{dy}{dx}=\dfrac{x^3(2x+5)-3x^2(x^2+5x-1)}{(x^3)^2}$

$=\dfrac{2x^4+5x^3-3x^4-15x^3+3x^2}{x^6}=\dfrac{-x^4-10x^3+3x^2}{x^6}=\dfrac{-x^2-10x+3}{x^4}$

b) $y=4x^3-7x+6\sqrt{x}-\dfrac{4}{x^3}$ $6x^{\frac{1}{2}}$ $-4x^{-3}$

$\dfrac{dy}{dx}=12x^2-7+6\left(\frac{1}{2}x^{-\frac{1}{2}}\right)+12x^{-4}$

$=12x^2-7+\dfrac{3}{\sqrt{x}}+\dfrac{12}{x^4}$

c) $y=\sin x\cdot\cos x$ Product Rule

$\dfrac{dy}{dx}=\sin x(-\sin x)+\cos x(\cos x)$

$=-\sin^2x+\cos^2x$

d) $y=\dfrac{\cot x}{x^3}$ Quotient Rule

$\dfrac{dy}{dx}=\dfrac{x^3(-\csc^2x)-\cot x(3x^2)}{(x^3)^2}$

$=\dfrac{-x^3\csc^2x-3x^2\cot x}{x^6}=\dfrac{-x\csc^2x-3\cot x}{x^4}$

e) $y=\dfrac{\sin x}{1+\tan x}$ Quotient Rule

$\dfrac{dy}{dx}=\dfrac{(1+\tan x)\cos x-\sin x(\sec^2x)}{(1+\tan x)^2}$

$=\dfrac{\cos x+\tan x\cos x-\sin x\sec^2x}{(1+\tan x)^2}$

OR

$=\dfrac{\cos x+\sin x-\tan x\sec x}{(1+\tan x)^2}$

f) $y=(1+\cos x)(1-\cos x)$ Product Rule

$\dfrac{dy}{dx}=(1+\cos x)(\sin x)+(1-\cos x)(-\sin x)$

$=\sin x+\cos x\sin x-\sin x+\cos x\sin x$

$=2\cos x\sin x$

4. Find the equation of the tangent line and normal line to $y = 4\cos x$ at $x = \frac{2\pi}{3}$.

Point $y = 4\cos\frac{2\pi}{3} = 4\left(-\frac{1}{2}\right) = -2$

$\left(\frac{2\pi}{3}, -2\right)$

slope $= y' = -4\sin x = -4\sin\frac{2\pi}{3} = -4\left(\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$

$y + 2 = -2\sqrt{3}\left(x - \frac{2\pi}{3}\right)$

5. Find $\frac{d^{101}y}{dx^{101}}$ of y = sinx.

$y' = \cos x$

$y'' = -\sin x$

$y''' = -\cos x$

$y^{iv} = \sin x$

$\frac{101}{4} = 25\frac{1}{4}$

Cosx

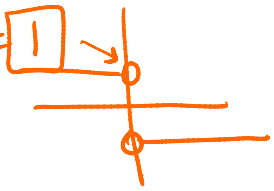
Calculator OK

Part 3: Chapter 2

1. Find the limit using the graph.

a. $\lim_{x\to 0^+} 2^{-\frac{1}{x}} =$ 0

b. $\lim_{x\to 0^-} \frac{|-x|}{-x} =$ 1



2. Find the average rate of change of $f(x) = x - 3x^2$ over the interval $[-4, -1]$.

AROC $= \frac{f(-1) - f(-4)}{-1 - (-4)} = \frac{-4 - (-52)}{3} =$ 16

3. For $f(x) = x - 3x^2$

a. Find the slope of the curve at x= -1. Show all your calculations using LIMITS.

$f'(x) = 1 - 6x$

$f'(-1) = 1 - 6(-1) = 7$

$f'(-1) = \lim_{h\to 0} \frac{(-1+h) - 3(-1+h)^2 - (-4)}{h}$

$= \lim_{h\to 0} \frac{-1 + h - 3 + 6h - 3h^2 + 4}{h}$

$= \lim_{h\to 0} \frac{7h - 3h^2}{h} = \lim_{h\to 0} 7 - 3h = 7$

b. Write the equation of the tangent to the curve at x= -1.

Point $(-1, -4)$

Slope = 7

$y + 4 = 7(x + 1)$

c. Write the equation of the normal to the curve at x= -1.

$\perp$ slope $= -\frac{1}{7}$

$y + 4 = -\frac{1}{7}(x + 1)$

Part 4: Chapter 3.1-3.5

1. On Earth, if you shoot a rubber band 64 feet straight up into the air, the rubber band will be $s(t) = 64t - 16t^2$ feet above your hand at t seconds after firing.

Position

a) Find v(t) and a(t).

$v(t) = s'(t) = 64 - 32t$

$a(t) = s''(t) = -32$

b) How long does it take the rubber band to reach its maximum height? (Use calculus methods.)

max height → $v(t) = 0$

$64 - 32t = 0$

$t = 2$ seconds

c) With what velocity does it leave your hand?

$v(0) = 64$ ft/s

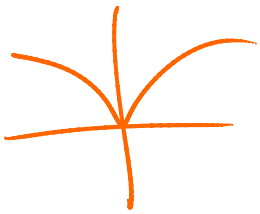
d) Is the rubber band speeding up, slowing down, or the same speed at t = 1 second? Justify.

$v(1) = 64 - 32(1) = 32$ $\quad v(1) > 0$

$a(1) = -32$ $\quad a(1) < 0$

Slowing down because velocity is positive but acceleration is negative

2. The function $f(x) = x^{\frac{4}{5}}$ is not differentiable at x = 0. What type of non-differentiability is it?



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