Chapter 7.2, 7.3 Review
Matrices

Name____________________________________

For 1-5, complete the matrix operation. If it is not possible, write "not possible". No Calculator.

1. $\begin{bmatrix} 5 & 6 & 1 & 0 \\ 2 & -2 & 3 & 4 \\ 1 & 5 & -2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 4 & -1 & 1 \\ -2 & 0 & 5 & 8 \\ 10 & -3 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 & -1 \\ 4 & -2 & -2 & -4 \\ -9 & 8 & -7 & -6 \end{bmatrix}$

times!

2. $4 \cdot \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 7 & 9 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 12 & -4 \\ 8 & 16 \end{bmatrix} + \begin{bmatrix} 7 & 9 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 19 & 5 \\ 7 & 17 \end{bmatrix}$

3. $\begin{bmatrix} 4 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} 5 & 3 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 20 & 12 & -4 & 8 \\ 25 & 15 & -5 & 10 \end{bmatrix}$

2x1 · 1x4 ✓

4. $\begin{bmatrix} 3 & 2 & -1 & 0 & 4 \\ 5 & 9 & -2 & -3 & 5 \\ 1 & 0 & -4 & -1 & 3 \\ 7 & 8 & 1 & 2 & 4 \\ 0 & -3 & 4 & -3 & 1 \\ 8 & 10 & -2 & 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 5 \\ 4 & -1 \end{bmatrix} =$ Not possible :)

6x5 No 2x2

5. $\begin{bmatrix} -1 & 2 & 0 \\ 4 & 1 & -2 \\ 3 & 7 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 \\ 0 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -10 & 1 \\ -6 & 9 \end{bmatrix}$

3x3 ✓ 3x2

3x2

For 6-8, determine if the inverse of the matrix exists. If it does exist, find it! (#6 and #7 No Calculator, #8 Calculator OK)

6. $\begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix}$ $\det A = 2(-2) - (-1)(4)$

$\det A = 0$

No inverse

7. $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ $\det A = 2(5) - 3(4) = -2$ Yes

$A^{-1} = \frac{1}{-2} \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{5}{2} & \frac{3}{2} \\ 2 & 1 \end{bmatrix}$

8. $\begin{bmatrix} 4 & -1 & 3 \\ 2 & 1 & 4 \\ 5 & -2 & 0 \end{bmatrix}$

$\begin{bmatrix} -.53 & .4 & .47 \\ -1.33 & 1 & .67 \\ .6 & -.2 & -.4 \end{bmatrix}$

9. Explain in words how you would prove that two matrices are inverses of each other.

① Multiply the 2 matrices together to see if their product is Identity matrix. $AB = BA = I$?

OR

② Find inverse of one matrix and see if it equals the other matrix. $A^{-1} = B$?

10. Explain in words what the "identity matrix" is.

A square matrix that, when multiplied by a matrix of the same size, gives the given matrix.

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $AI = A$ $IA = A$

For 11-13, solve the system of equations using matrices (Calculator OK). You must use each method at least once (Inverses and Reduced Row Echelon Form).

11. $2x - 3y = -10$
$x + 2y = 16$

$$\begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -10 \\ 16 \end{bmatrix}$$

$A \cdot X = B$

$X = A^{-1}B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$x = 4$ $y = 6$

12. $x + y + z = 2$
$2x - 3y + z = -5$
$3x + 2y + 4z = 3$

$$\text{RREF}\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & -3 & 1 & -5 \\ 3 & 2 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

So $x = 1$ $y = 2$ $z = -1$

13. $x + y + z = -2$
$2x + z = -1$
$3y + 3z = -12$

$$\text{RREF}\begin{bmatrix} 1 & 1 & 1 & -2 \\ 2 & 0 & 1 & -1 \\ 0 & 3 & 3 & -12 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -5 \end{bmatrix}$$

$x = 2$ $y = 1$ $z = -5$

$$x=2 \quad y=1 \quad z=-5$$

14. Mrs. Billz has paper money in her wallet consisting of \$1 bills, \$5 bills, \$10 bills, and \$20 dollar bills. On Friday she had 19 total bills in her wallet that adds up to \$125. She also has one more \$10 bill than the total number of \$5 bills. The number of \$20 bills is equal to the number of \$5 bills minus the number of \$1 bills. How many of each type of bill does she have?

x = # of \$1 bills
y = # of \$5 bills
z = # of \$10 bills
w = # of \$20 bills

$x + y + z + w = 19$: total # bills
$1x + 5y + 10z + 20w = 125$: total \$
$z - 1 = y$: one more \$10 than \$5s
$w = y - x$: \$20s = \$5s minus \$1s

$0 - y + z + 0 = 1$
$x - y + 0 + w = 0$

$$\text{RREF}\begin{bmatrix} 1 & 1 & 1 & 1 & 19 \\ 1 & 5 & 10 & 20 & 125 \\ 0 & -1 & 1 & 0 & 1 \\ 1 & -1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

5 \$1 bills
6 \$5 bills
7 \$10 bills
1 \$20 bills

15. The Gaussians Math team has made it to State! After they compete at State, they take home a total of 10 trophies (1^{st}, 2^{nd}, and 3^{rd} place finishes in each event earns the team a trophy). The number of 1^{st} place trophies is the same as the number of 2^{nd} and 3^{rd} place trophies combined. Also, the number of 1^{st} place trophies is one less than twice the number of 2^{nd} place trophies. How many 1^{st}, 2^{nd} and 3^{rd} place trophies do they take home?

a = # of 1^{st} place trophies
b = # of 2^{nd} place trophies
c = # of 3^{rd} place trophies

$a + b + c = 10$
$a = b + c \Rightarrow a - b - c = 0$
$a = 2b - 1 \qquad a - 2b + 0 = -1$

$$\text{RREF}\begin{bmatrix} 1 & 1 & 1 & 10 \\ 1 & -1 & -1 & 0 \\ 1 & -2 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

5 1^{st} place trophies
3 2^{nd} place trophies

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 1 & -2 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

3 2^{nd} place trophies
2 3^{rd} place trophies