The function $m(t)$

models the rate at which the population of Mathtown is growing (in people per year), where *t* is measured in years since January 1st, 2010.

If the population of Mathtown was 30,000 people at the beginning of 2012,

Write an expression involving an integral for the population of the city at the beginning of 2016.

$$M(6) = M(2) + \int_2^6 m(t)\,dt$$

$$= 30{,}000 + \int_2^6 m(t)\,dt$$

At a certain height, a tree trunk has a circular cross section. The radius $R(t)$ of that cross section grows at a rate modeled by the function

$$R'(t) = \frac{dR}{dt} = \frac{1}{16}\left(3 + \sin\left(t^2\right)\right) \text{ centimeters per year}$$

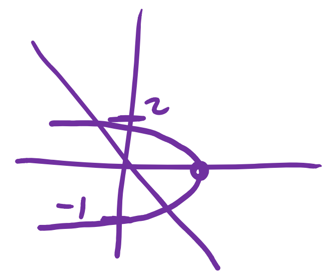
for $0 \le t \le 3$, where time t is measured in years. At time $t = 0$, the radius is 6 centimeters. The area of the cross section at time t is denoted by $A(t)$. $R(0) = 6$

(a) Write an expression, involving an integral, for the radius $R(t)$ for $0 \le t \le 3$. Use your expression to find $R(3)$.

$$R(3) = R(0) + \int_0^3 R'(t)\,dt$$

$$= 6 + \int_0^3 R'(t)\,dt = \boxed{6.611 \text{ cm}}$$

Find the area enclosed by the curves.
NO CALCULATOR!!



$$x = 2 - y^2 \;\; and \;\; x = -y$$

$2 - y^2 = -y$

$y^2 - y - 2 = 0$

$(y+1)(y-2) = 0$

$y = -1, 2$

$A = \int_{-1}^{2} (2 - y^2 - (-y))\,dy = \int_{-1}^{2} (2 - y^2 + y)\,dy$

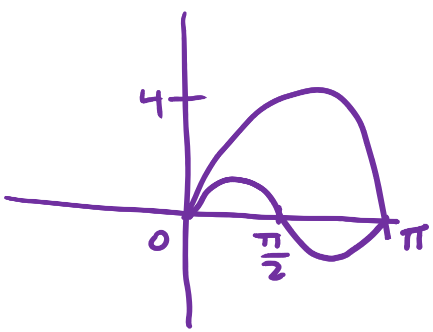
$= 2y - \frac{1}{3}y^3 + \frac{1}{2}y^2 \Big|_{-1}^{2} = 2(2) - \frac{1}{3}(2)^3 + \frac{1}{2}(2)^2$

$- (2(-1) - \frac{1}{3}(-1)^3 + \frac{1}{2}(-1)^2)$

$= \boxed{9/2}$

Find the area enclosed by the curves.
NO CALCULATOR!!

$$y = 4\sin x \ \ and \ \ y = \sin 2x \quad 0 \le x \le \pi$$



$A = \int_0^{\pi} (4\sin x - \sin 2x)\,dx = -4\cos x + \frac{1}{2}\cos 2x \Big|_0^{\pi}$

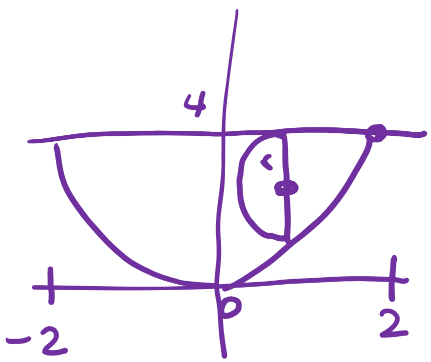
$= -4\cos\pi + \frac{1}{2}\cos 2\pi$

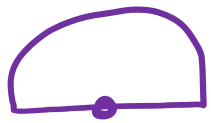
$- (-4\cos 0 + \frac{1}{2}\cos 2\cdot 0)$

$= \boxed{8}$

The base of a region is between the line y=4 and the parabola y = x^2.

. The cross sections of the solid are perpendicular to the x-axis and are semi-circles. Find the volume.





$d = 4 - x^2$

$r = \dfrac{4-x^2}{2}$

$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\dfrac{4-x^2}{2}\right)^2$

$= \frac{1}{2}\pi\left(\dfrac{16-8x^2+x^4}{4}\right)$

$A = \frac{1}{8}\pi\left(16-8x^2+x^4\right)$

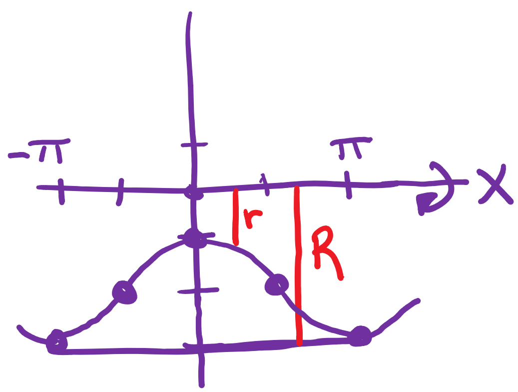
$V = \int_{-2}^{2} \frac{1}{8}\pi\left(16-8x^2+x^4\right)dx$

$= \frac{1}{8}\pi\left(16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right)\Big|_{-2}^{2}$

$= \dfrac{64\pi}{15} \approx \boxed{13.404}$

A region bounded by $y = \cos x - 2,\ y = -3,\ and\ [-\pi, \pi]$

Find the volume of the solid if it is rotated about the x-axis.



$R = 3 - 0 = 3$

$r = \cos x - 2 - 0$

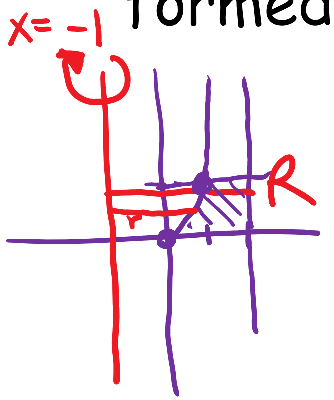
$r = \cos x - 2$

$V = \int_{-\pi}^{\pi} \left(\pi 3^2 - \pi(\cos x - 2)^2\right) dx$

$\approx \boxed{88.826}$

Let R be the region enclosed by

$y = x^3$, y = 1, x = 2, and the x-axis. Find the volume of the solid formed by revolving the region about the line x = -1.



$R = -1 - 2 = -3$

$r = -1 - \sqrt[3]{y}$

$y = x^3$

$x = \sqrt[3]{y}$

$$V = \int_0^1 \left(\pi(-3)^2 - \pi\left(-1 - \sqrt[3]{y}\right)^2\right) dy \approx \boxed{18.535}$$