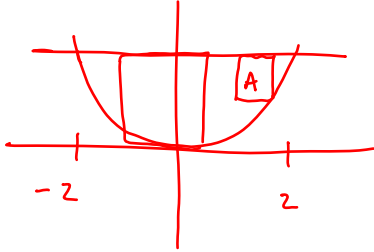


1. The base of a region between the line  $y=4$  and the parabola  $y=x^2$ . The cross sections of the solid are perpendicular to the x-axis are:

a. Squares:

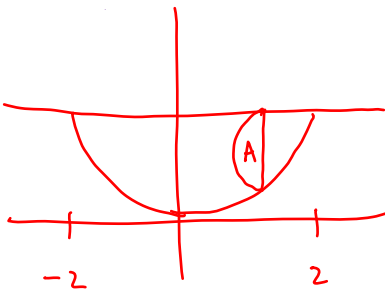
$$A(x) = (4 - x^2)^2 = 16 - 8x^2 + x^4$$



$$V = 2 \int_0^2 (16 - 8x^2 + x^4) dx = \frac{512}{15} \approx 34.133$$

b. Semicircles:

$$A(x) = \frac{1}{2} \pi \left( \frac{4 - x^2}{2} \right)^2 = \frac{\pi}{8} (4 - x^2)^2 = \frac{\pi}{8} (16 - 8x^2 + x^4)$$

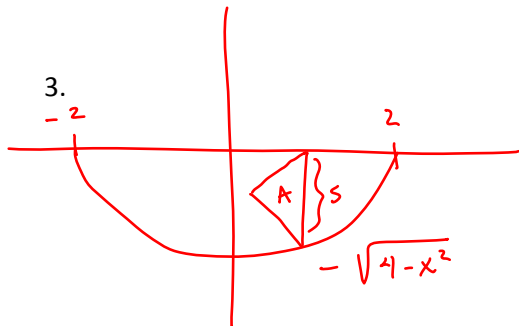


$$V = 2 \int_0^2 \frac{\pi}{8} (16 - 8x^2 + x^4) dx = \frac{\pi}{8} \cdot \frac{512}{15} = \frac{64\pi}{15} \approx 13.404$$

2. Find the volume of the solid that lies between planes perpendicular to the x-axis at  $x=2$  and  $x=-2$ . The cross sections perpendicular to the x-axis between the planes are equilateral triangles whose bases run from  $y=0$  to  $y=-\sqrt{4-x^2}$ .

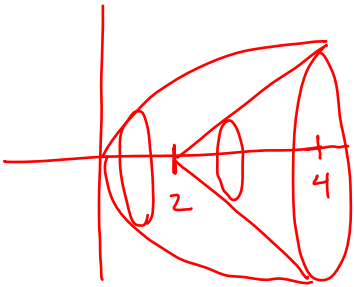
$$s = (0 - (-\sqrt{4-x^2})) = \sqrt{4-x^2}$$

$$A(x) = \frac{s^2}{4} \sqrt{3} = \frac{4-x^2}{4} \sqrt{3}$$



$$V = 2 \int_0^2 \frac{\sqrt{3}}{4} (4-x^2) dx = \frac{\sqrt{3}}{2} \cdot \frac{16}{3} = \frac{16\sqrt{3}}{3} \approx 4.619$$

A region is bounded by  $y = \sqrt{x}$ ,  $y = x - 2$ , and  $y = 0$ . What is the volume of the solid generated when this region is rotated around the x-axis?

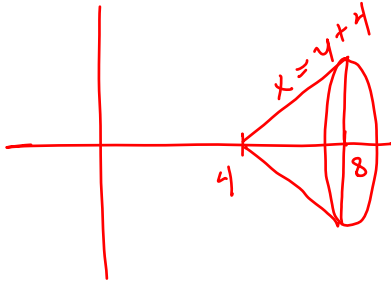


$$V = \int_0^2 \pi (\sqrt{x})^2 dx + \int_2^4 \pi [(\sqrt{x})^2 - (x-2)^2] dx$$

$$= 2\pi + \frac{10}{3}\pi = \frac{16}{3}\pi$$

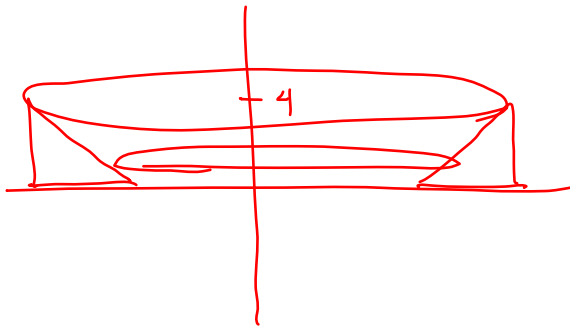
4. A region is bounded by  $f(y) = 4 + y$ , and  $[0, 4]$ . Find the volume of the solid if it is rotated about:

a. The x-axis



$$\int_4^8 \pi (x-4)^2 dx = \frac{64\pi}{3} \approx 67.021$$

b. The y-axis

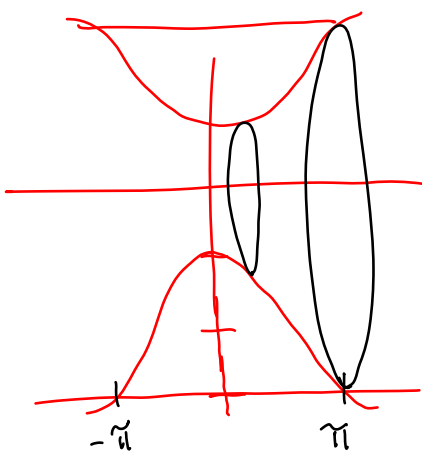


$$A(y) = \pi (8^2 - (4+y)^2)$$

$$V = \int_0^4 \pi (8^2 - (4+y)^2) dy = \frac{320\pi}{3}$$

$$\approx 335.103$$

5. A region bounded by  $y = \cos x - 2$ ,  $y = -3$ , and  $[-\pi, \pi]$ . Find the volume of the solid if it is rotated about the x-axis.



$$A(x) = \pi [(-3)^2 - (\cos x - 2)^2]$$

$$V = 2\pi \int_0^{\pi} [(-3)^2 - (\cos x - 2)^2] dx = \frac{103,713}{3678} \pi$$

$$\approx 88.824$$