9.2 Notes - l'Hospital's Rule

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Find the limit:

① $\lim\limits_{t\to 2} \dfrac{t^2-3t+2}{t^2-4} = \dfrac{0}{0}$

$= \lim\limits_{t\to 2} \dfrac{(t-1)\cancel{(t-2)}}{(t+2)\cancel{(t-2)}} = \lim\limits_{t\to 2} \dfrac{t-1}{t+2}$

$= \dfrac{2-1}{2+2} = \boxed{\dfrac{1}{4}}$

② $\lim\limits_{x\to 0} \dfrac{3x+\sin x}{x} = \dfrac{0}{0}$

$= \lim\limits_{x\to 0} \dfrac{3\cancel{x}}{\cancel{x}} + \lim\limits_{x\to 0} \dfrac{\sin x}{x}$

$= 3 + 1 = \boxed{4}$

③ $\lim\limits_{x\to 0} \dfrac{\sin x}{2x^2-x} = \dfrac{0}{0}$

$= \lim\limits_{x\to 0} \dfrac{\sin x}{x(2x-1)}$

$= \lim\limits_{x\to 0} \dfrac{\sin x}{x} \cdot \dfrac{1}{2x-1}$

$= 1 \cdot \dfrac{1}{2(0)-1} = \boxed{-1}$

④ $\lim\limits_{x\to 1} \dfrac{\ln x}{x^2-1} = \dfrac{0}{0}$

hmmm

⑤ $\lim\limits_{\theta\to \frac{\pi}{2}} \dfrac{\cos\theta}{\frac{\pi}{2}-\theta} = \dfrac{0}{0}$

hmmm.

l'Hôspital's Rule for Evaluating Limits of Indeterminate Forms $\dfrac{0}{0}$, $\dfrac{\infty}{\infty}$

If $f(a)=g(a)=0$ or $f(a)=g(a)=\infty$ or $-\infty$, and $f'(a)$, $g'(a)$ exist:

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$$

④ $\lim\limits_{x\to 1} \dfrac{\ln x}{x^2-1} = \dfrac{0}{0}$ Indet. Form, so L.R. applies

$\lim\limits_{x\to 1} \dfrac{\ln x}{x^2-1} = \dfrac{\frac{1}{1}}{2(1)} = \boxed{\dfrac{1}{2}}$ (see Graph)

⑤ $\lim\limits_{\theta\to \frac{\pi}{2}} \dfrac{\cos\theta}{\frac{\pi}{2}-\theta} = \dfrac{0}{0}$ indet. Form, so L.R. applies.

$= \dfrac{-\sin\frac{\pi}{2}}{-1} = \dfrac{-1}{-1} = \boxed{1}$

Caution! If limit does not result in Indeterminate Form, L.R. will not work!

Ex: $\lim\limits_{x\to 3} \dfrac{x-3}{x^2-4} = \dfrac{0}{5} = \boxed{0}$ ← answer

If L.R. applied: $\dfrac{1}{2(3)} = \dfrac{1}{6}$ ∴ wrong!

Try Some!

① $\lim \sqrt{1+x} - 1$ ...

② $\lim \ln x$ ... $= \dfrac{\infty}{\infty}$ L.R.

Try Some!

① $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{x} = \dfrac{0}{0}$ L.R. applies

$= \dfrac{\frac{1}{2}(1+0)^{-\frac{1}{2}}}{1} = \boxed{\frac{1}{2}}$

② $\lim\limits_{x\to \infty} \dfrac{\ln x}{2\sqrt{x}} = \dfrac{\infty}{\infty}$ L.R. Applies

$= \dfrac{\frac{1}{\infty}}{\cancel{2}\cdot\frac{1}{\cancel{2}}(\infty)^{-\frac{1}{2}}} = \dfrac{\frac{1}{\infty}}{\frac{1}{\sqrt{\infty}}} = \dfrac{1}{\infty}\cdot\sqrt{\infty} = \dfrac{1}{\sqrt{\infty}} = \boxed{0}$

Sometimes, $\dfrac{f'(a)}{g'(a)}$ is still Indeterminate $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$.

If so, you can try to repeat l'Hôsptital's Rule.

l'Hôspital's Rule (Stronger Form)

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)} \cdots$$

Examples

③ $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2} = \dfrac{0}{0}$ L.R. applies

$= \lim\limits_{x\to 0} \dfrac{\sin x}{2x} = \dfrac{0}{0}$ L.R. applies

$= \lim\limits_{x\to 0} \dfrac{\cos x}{2} = \boxed{\frac{1}{2}}$

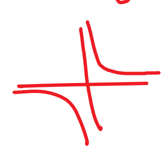
Using l'Hôspital's Rule to Evaluate L/R-Hand Limits - Yes You can!

Use Stronger Form.

④ $\lim\limits_{x\to 0^-} \dfrac{\sin x}{x^2} = \dfrac{0}{0}$ LR

$= \lim\limits_{x\to 0^-} \dfrac{\cos x}{2x} = \dfrac{1}{-0} = -\infty$

Graph of $y = \dfrac{\sin x}{x^2}$



⑤ $\lim\limits_{x\to 0^+} \dfrac{\sin x}{x^2} = \dfrac{0}{0}$ L.R.

$= \lim\limits_{x\to 0^+} \dfrac{\cos x}{2x} = \dfrac{1}{0} = \infty$

$\dfrac{0}{0}, \dfrac{\infty}{\infty}$ = Indeterminate

$\dfrac{\#}{\infty}, \dfrac{0}{\infty}, \dfrac{0}{\#} = 0$

$\dfrac{\infty}{\#}, \dfrac{\#}{0} = \infty$

[Also $\infty\cdot 0, \infty-\infty, 1^{\infty}, 0^0, \infty^0$]

$x \to 0$

$$\boxed{\begin{array}{c} \frac{0}{0},\ \frac{\infty}{\infty} = \text{Indeterminate} \\ \frac{\#}{\infty},\ \frac{0}{\infty},\ \frac{0}{\#} = 0 \\ \frac{\infty}{\#},\ \frac{\#}{0} = \infty \end{array}}$$

[Also $\infty \cdot 0$, $\infty - \infty$, 1^{∞}, 0^{0}, ∞^{0}]