9.1 Notes Day 1

Thursday, May 7, 2015
3:38 PM

Precalculus
Section 9.1 Notes – Day 1

Combinatorics

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PRETEST: How many of these questions can you answer on your own?

1. How many different ways can you order the letters ABC?

6

ABC
ACB
BAC
BCA
CBA
CAB

2. Four people are running for office. How many different ways can they be chosen to be the President, VP, Secretary, and Treasurer?

$\underset{P}{\underline{4}} \cdot \underset{VP}{\underline{3}} \cdot \underset{S}{\underline{2}} \cdot \underset{T}{\underline{1}}$ $\boxed{24}$

3. How many different 6 – letter "words" can you make out of the word FRIDAY?

$\underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = \boxed{720}$

4. You have 4 shirts and 3 pairs of shorts. How many different outfits can you make?

$4 \cdot 3 = \boxed{12}$

5. 8 people are trying out for 3 different roles in a school play. How many different ways may the 3 roles be cast?

$\underline{8} \cdot \underline{7} \cdot \underline{6} = \boxed{336}$

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Multiplication Principle of Counting

If a task consists of a sequence of choices in which there are *p* selections for the first choice, *q* selections for the second choice, *r* selections for the third choice, and so on, then the total number of selections possible is determined by:

$p \cdot q \cdot r$ (product of choices)

Math Selfies! You are at a math party and would like to take a bunch of trig selfies to post on Facebook. How many selfies will you need to take if you wish to have a selfie that includes you and each of: 4 friends, 3 backgrounds, and a trigonometric function?

$\underset{\text{people}}{\underline{4}} \cdot \underset{\text{backgr}}{\underline{3}} \cdot \underset{\text{trig functions}}{\underline{6}} = \boxed{72}$

Airport Codes Since the 1930s, airports have been assigned three-letter codes which are prominently displayed on baggage tags attached at airport check-in desks. The standard naming convention is to use the first three letters of the city name in which the airport is located. (Of course, there are many cases in which this doesn't hold true. For instance, cities that have two airports, cities in different states that have the same name, all Canadian airport codes begin with "Y", and so on ☺) How many three-letter airport codes are possible?

$26 \cdot 26 \cdot 26 = 26^3 = 17{,}576$

(letters can be repeated)

Call Me, Maybe! Initial use of three-digit area codes in the United States and Canada began in 1947 in large cities. Without any restrictions, how many three-digit area codes are possible?

10 digits total [0-9]

$10 \quad 10 \quad 10 = 10^3 = 1000$

At first, area codes were all in the form *NYX*, where *N* is any integer 2 through 9, *Y* is 0 or 1, and *X* is any integer 1 through 9 (if *Y* is 0) or any integer 2 through 9 (if *Y* is 1). (In other words, 312 was okay, but 311 was not.)

With these restrictions, how many area codes were possible?

$\underset{N}{8} \cdot \underset{Y}{1} \cdot \underset{X}{9} = 72$

$+ \qquad +$

$\underset{N}{8} \quad \underset{Y}{1} \quad \underset{X}{8} = 64$

136

Why the restrictions? The restriction on *N* saves 0 for calling the operator, and 1 for signaling a long-distance call. The restriction on the second digit, limiting it to 0 or 1, was designed to help telephone equipment recognize the difference between a three-digit "area code" (with 0 or 1 as the second digit) and the three-digit "exchange" prefix (which had avoided 0 or 1 for the second digit, because of restrictions in existing switching equipment).

In the 1990s, some of the restrictions were lifted. Since that time, area codes allow for the first number to be any integer 2 through 9, and the second and third numbers to be any integer 0 through 9.

How many area codes are now possible?

$8 \cdot 10 \cdot 10 = 800$

License and Registration, Please Currently, non-personalized Illinois license plates come in two forms:

a. 7-digits such that the first and last digits may be any integer 1 through 9, and the middle five digits may be any integer 0 through 9. How many of these plates are possible?

$9 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10 \quad 9 = 8{,}100{,}000$

b. A letter (A, J, X, B, C, D, F, T, Y, G, H, K, L, N, P, R, S, or V) followed by 6-digits such that each digit may be any integer 0 through 9. How many of these plates are possible?

$\underset{L}{18} \quad 10 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10 = 18{,}000{,}000$

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Factorial Notation: The factorial of non-negative integer is written as $n!$ and represents the product of all positive integers less than or equal to n.

Examples:

$3! = 3\cdot 2\cdot 1 = 6$

$6! = 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 = 720$

$\frac{6!}{4!} = \frac{6\cdot 5\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}{\cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}} = 6\cdot 5 = 30$

$0! = 1$ (Def)

Permutation: The act of permuting objects refers to counting the number of ways that a set of n objects can be arranged in order. Each such ordering is called a permutation of the set.

Smile! How many ways can a group of 5 people line up for a photograph?

$\underline{5}\cdot\underline{4}\cdot\underline{3}\cdot\underline{2}\cdot\underline{1} = 5! = 120$

How many ways can a group of 5 people (a dad and his 4 kids) line up for a photograph if the dad must be in the middle?

$\underline{4}\cdot\underline{3}\cdot\underset{D}{\underline{1}}\cdot\underline{2}\cdot\underline{1} = 24$

Words, Words, Words: How many 6-letter "words" can be formed from the letters in the word PRECAL ?

$\underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 6! = 720$

Distinguishable Permutations What happens to the number of unique orderings of a set of objects if some of the objects are identical? ZZ and ZZ are same: divide by n! for repeated letters

Example: How many "words" can be formed from the letters in each of the following:

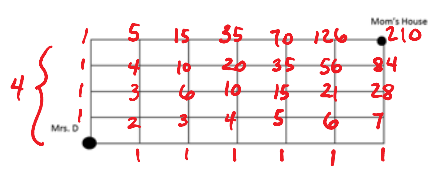
a. PIZZA b. BANANA (3 A's, 2 N's) c. SUCCESS (2 C's, 3 S's) d. Make up your own!

a. $\frac{5!}{2!} = \frac{5\cdot 4\cdot 3\cdot \cancel{2}\cdot \cancel{1}}{\cancel{2}\cdot \cancel{1}} = 60$

b. $\frac{6!}{3!\,2!} = \frac{6\cdot 5\cdot 4\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}{\cancel{3}\cdot \cancel{2}\cdot \cancel{1}\cdot 2\cdot 1} = 60$

c. $\frac{7!}{2!\,3!} = \frac{7\cdot 6\cdot 5\cdot 4\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}{\cancel{3}\cdot \cancel{2}\cdot \cancel{1}\cdot 2\cdot 1} = 420$

Happy Mother's Day! Mrs. D is travelling from home to visit her mother. If she travels only north and east, how many possible paths could she take?



OR $\frac{10!}{4!\,6!} = 210$

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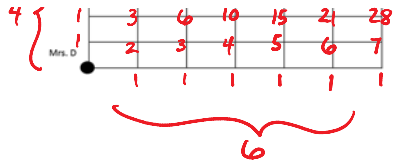

How many possible paths can Mrs. D take if she must stop at the flower shop on the way?

$6 \cdot 15 = 90$