

# Permutations and Combinations!

If you are arranging the letters A through G in the alphabet, how many different ways are there of doing this with no repetition?

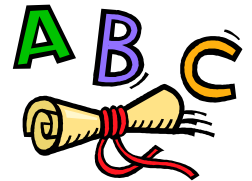
A B C D E F G 7 letters  
 $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

Another way of saying this is 7!

Seven Factorial MATH → PROB → 4

From these letters, how many 3 letter words can you make not repeating any letters?

$7 \times 6 \times 5 = 210$  7 letters total - choosing 3



This is called a permutation.

**Notation:**  ${}_n P_r$  order matters

**Formula:**  ${}_n P_r = \frac{n!}{(n-r)!}$   $n = \# \text{ elements in set}$   
 $r = \# \text{ elements chosen}$

1. For my softball team, I have 15 players. How many different ways can I pick a pitcher, then a catcher, a shortstop and then a center fielder? (Order matters b/c it must be in this order!)

${}_{15} P_4 = \frac{15!}{(15-4)!} = \frac{15!}{11!} = \underline{15} \cdot \underline{14} \cdot \underline{13} \cdot \underline{12} = 32,760$

2. How many 5 letter "words" can I make from the letters in **HINSDALE**?

No repetition  ${}_{8} P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!} = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 6720$

3. A permutation is selected at random from the letters **MISSISSIPPI**.

a) What is different about this word? Repetition

4 I's 4 S's 2 P's

b) How many different ways are there of arranging all the letters?

$\frac{11!}{4!4!2!} = 34,650$

**Permutations with Repetition**

$a, b \# \text{ repetitions}$   $\frac{{}_n P_r}{{}_a P_a \cdot {}_b P_b \dots}$

How many ways can 2 students from our class of 20 be chosen to go to the office?

$$\underline{20} \cdot \underline{19} \div 2 = 190 \quad \text{order doesn't matter, so choosing Alex then Allie is same as choosing Allie then Alex.}$$

This is called a Combination.

Notation:

$${}_n C_r$$

Formula:

$${}_n C_r = \frac{{}_n P_r}{r!} = \frac{n!}{r!(n-r)!}$$

4. In how many different ways can you form a committee of 5 people from a group of 9 people?

NOTE: Does order matter – permutation or combination?

No - just committee - not Pres, VP, etc!

$${}_9 C_5 = \frac{9!}{5!(4!)} = \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} = \text{126 ways}$$

$$\text{OR } \underline{9} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} \div 5!$$

$$= n(S)$$

5. A standard deck of playing cards has 52 cards. How many 5-card poker hands can be dealt from the deck?

$${}_{52} C_5 = \frac{52!}{5!(47!)} = 25,989,600$$



6. For the annual MathRules party, Mrs. D is buying treats! At the store, she finds 7 varieties of soda and 10 varieties of snacks. How many combinations of 3 soda options and 4 snack options are possible?

$${}_7 C_3 \cdot {}_{10} C_4 = 35 \cdot 210 = 7,350$$

7. A standard deck has 4 suits of 13 cards each. The suits are hearts, diamonds, spades, and clubs. If 5 cards are drawn, how many different combinations of 2 hearts, 2 clubs, and 1 diamond are possible?

$${}_{13} C_2 \cdot {}_{13} C_2 \cdot {}_{13} C_1 = 78 \cdot 78 \cdot 13 = 79,092$$

Think about your gym lock...

Should it be called a Locker Combination? Permutation! (order matters)