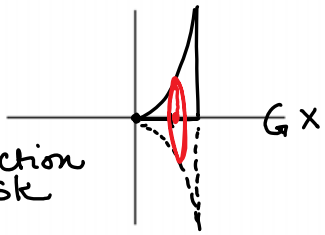
8.3 Rev Day 1

Wednesday, March 15, 2017 7:45 AM

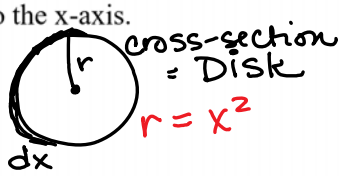
8.3 Solids of Revolution

1. The region bounded by $f(x) = x^2$, the x-axis, and x=2 is rotated about the x-axis.



a) **Picture** the solid of revolution that is formed.
b) Draw and label a slice made perpendicular to the x-axis. cross section
c) Find the area of a cross section of the solid.

$A = \pi r^2$ $A = \pi(x^2)^2$ $A = \pi(x)^4$



d) Find the volume of one slice.

$V_{\text{cross-section}} = \pi(x)^4 dx$

e) Give the volume of the solid as a definite integral.

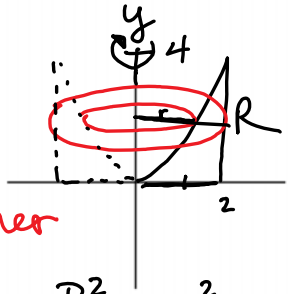
$$V = \int_0^2 \pi x^4 dx$$

f) Evaluate the integral to find the volume of the solid.

$V = \frac{1}{5}\pi x^5 \Big|_0^2 = \frac{1}{5}\pi(2)^5 - \frac{1}{5}\pi(0)^5$

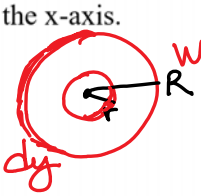
$= \frac{32\pi}{5}$

2. The region bounded by $f(x) = x^2$, the x-axis , and x=2 is rotated about the y-axis. $y = x^2$ $x = \pm\sqrt{y}$



a) **Picture** the solid of revolution that is formed.
b) Draw and label a slice made perpendicular to the x-axis.
c) Find the area of a cross section of the solid.

$R = 2$
$r = \sqrt{y}$



$A = \pi R^2 - \pi r^2$
$A = \pi(4) - \pi(\sqrt{y})^2$
$A = \pi(4 - y)$

d) Find the volume of one slice.

$V = \pi(4 - y)dy$ cross-section

e) Give the volume of the solid as a definite integral.

$$V = \int_0^4 \pi(4 - y)\,dy$$

f) Evaluate the integral to find the volume of the solid.

$= \pi(4y - \frac{1}{2}y^2)\Big|_0^4 = \pi 4(4) - \frac{1}{\pi 2}(4)^2 - (\pi(4)(0) - \frac{1}{\pi 2}(0)^2)$

$= 16\pi - 8\pi$

$= 8\pi$