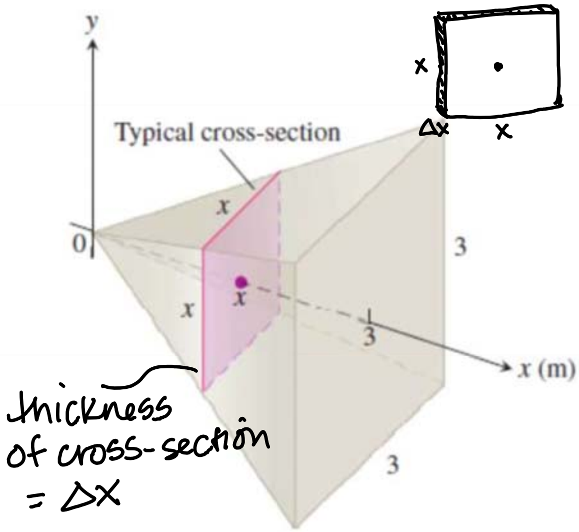
8.3 Day 1 Notes

Thursday, March 9, 2017 2:32 PM

AP Calculus AB 8.3 Volume



thickness of cross-section $= \Delta x$

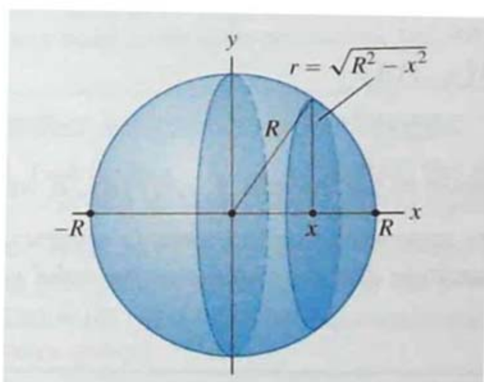
Area of cross-section $= A(x_k)$

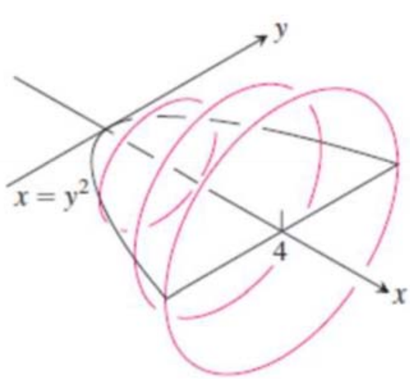
$$\text{VoL} = \sum A(x_k)\cdot \Delta x$$

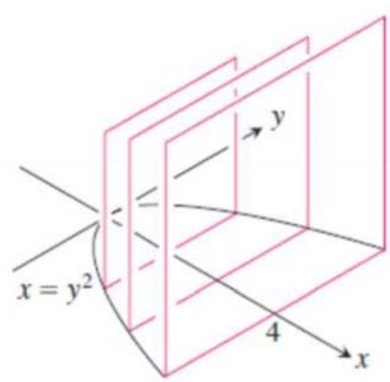
$$\boxed{V = \int_a^b A(x)\,dx}$$

= volume of a solid w/ cross-section Area $A(x)$ from $x=a$ to $x=b$

Find the volume of a sphere of radius R.



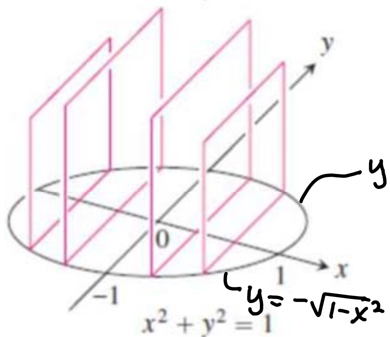


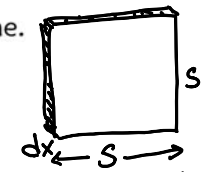



4
x

Find the volume of the solid created. The solids lay between planes perpendicular to the x-axis at x = -1 and x = 1. The cross sections perpendicular to the x-axis between these planes run from the semi-circle $y=-\sqrt{1-x^2}$ to the semi-circle $y=\sqrt{1-x^2}$.

a) Cross sections are squares with bases in the xy-plane.





$\leftarrow S \rightarrow$

$S=\sqrt{1-x^2}-(-\sqrt{1-x^2})$

$S=2\sqrt{1-x^2}$

$A=S^2$

$A=(2\sqrt{1-x^2})^2$

$A=4(1-x^2)=4-4x^2$

$$\boxed{V=\int_{-1}^{1}(4-4x^2)\,dx}$$

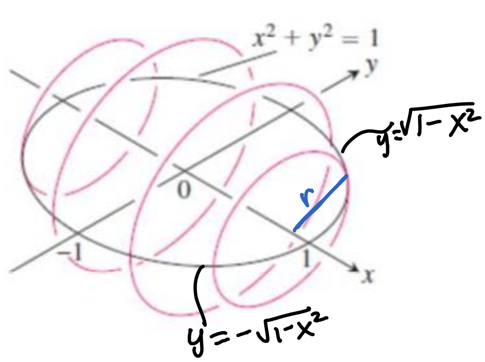
$$=4x-\frac{4}{3}x^3\Big|_{-1}^{1}$$

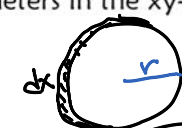
$$=4(1)-\frac{4}{3}(1)^3-\left(4(-1)-\frac{4}{3}(-1)^3\right)$$

$$=4-\frac{4}{3}+4-\frac{4}{3}$$

$$=\boxed{\frac{16}{3}}$$

b) Cross sections are circular disks with diameters in the xy-plane.





$r=\sqrt{1-x^2}$

$A=\pi r^2$

$A=\pi(\sqrt{1-x^2})^2$

$A=\pi(1-x^2)$

$$V=\int_{-1}^{1}\pi(1-x^2)\,dx$$

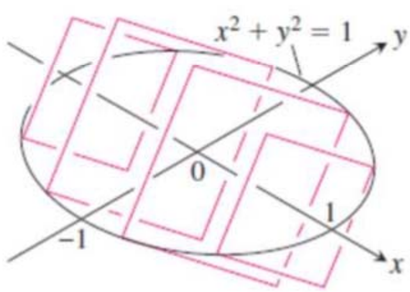
$$=\pi\left(x-\frac{1}{3}x^3\right)\Big|_{-1}^{1}$$

$$=\pi\left[\left(1-\frac{1}{3}(1)^3\right)-\left((-1)-\frac{1}{3}(-1)^3\right)\right]$$

$$=\pi\left(\frac{2}{3}+\frac{2}{3}\right)$$

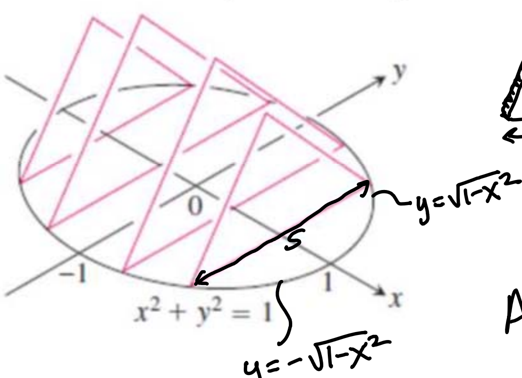
$$=\boxed{\frac{4\pi}{3}}$$

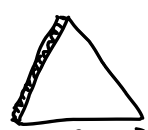
c) Cross sections are squares with diagonals in the xy-plane.



d) Cross sections are equilateral triangles with bases in the xy-plane.

$A_{eq\Delta}=\frac{\sqrt{3}s^2}{4}$





$\leftarrow S \rightarrow$

$S=2\sqrt{1-x^2}$

$A=\frac{\sqrt{3}(2\sqrt{1-x^2})^2}{4}$

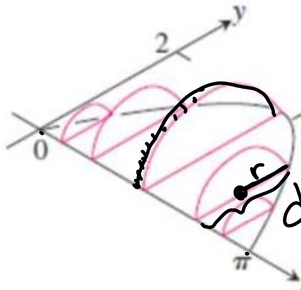
$A=\frac{\sqrt{3}(\cancel{4}(1-x^2))}{\cancel{4}}$

$A=\sqrt{3}(1-x^2)$

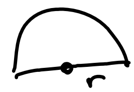
$$\boxed{V=\int_{-1}^{1}\sqrt{3}(1-x^2)\,dx}$$

≈ 2.309

Find the volume of the solid created so its base is the shape of the region between the x-axis and one arch of the curve $y = 2\sin x$. Each cross section cut perpendicular to the x-axis is a semi-circle whose diameter runs from the x-axis to the curve.



$d = 2\sin x$
$r = \sin x$



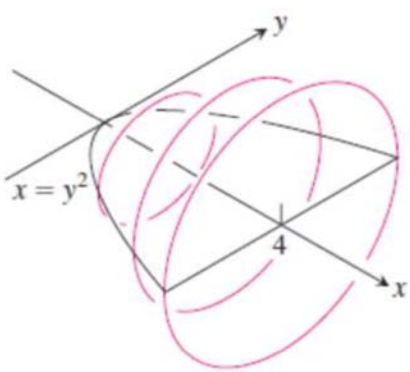
$A = \frac{1}{2}\pi r^2$

$A = \frac{1}{2}\pi \sin^2 x$

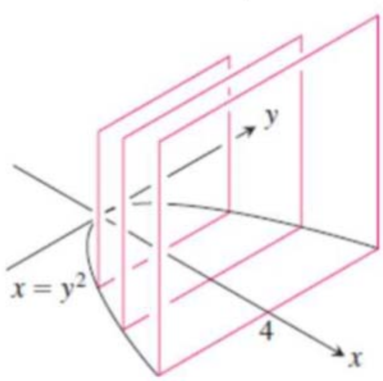
$V = \int_0^{\pi} \frac{1}{2}\pi \sin^2 x \, dx \approx \boxed{2.467}$

The solid lies between planes perpendicular to the x-axis at x = 0 and x = 4. The cross sections perpendicular to the x-axis between these planes run from $y = -\sqrt{x}$ to $y = \sqrt{x}$.

a) The cross sections are circular disks with diameters in the xy-plane.



b) The cross sections are squares with bases in the xy-plane.



* Find volume of solid:
Between y-axis, line $y = 3$, $y = \sqrt{x}$

(a) cross-sections are eq Δ $\perp$ x-axis
(b) cross-sections are squares $\perp$ to y-axis