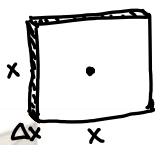
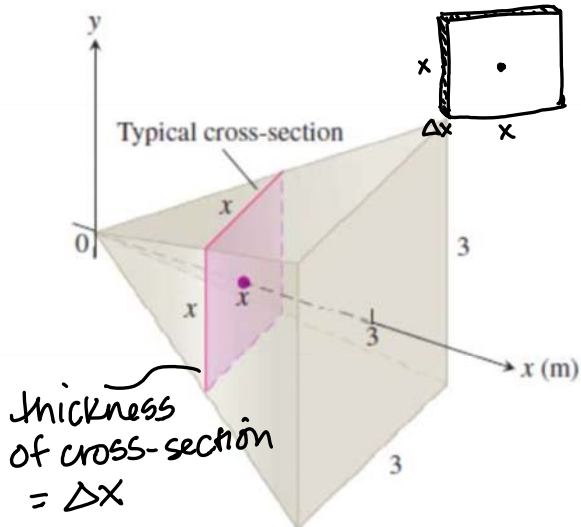


8.3 Day 1 Notes

Thursday, March 9, 2017 2:32 PM

AP Calculus AB

8.3 Volume



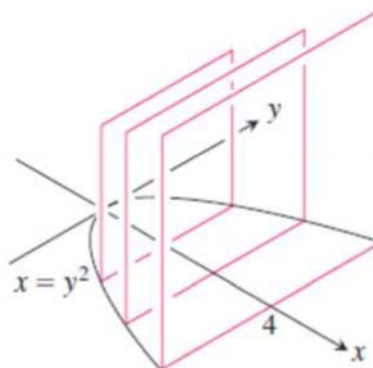
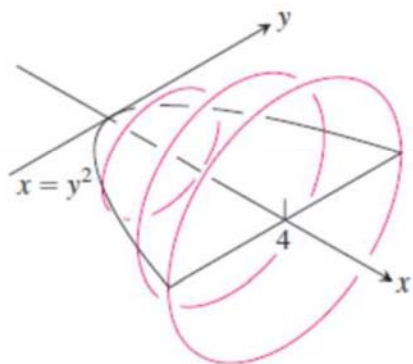
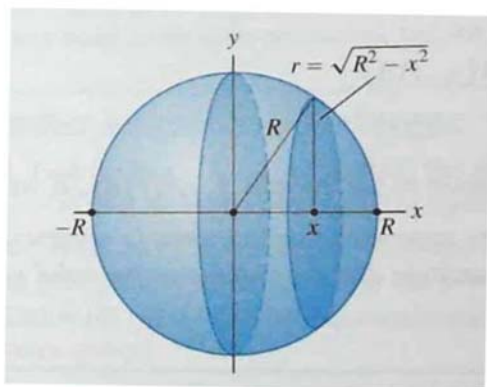
Area of cross-section = $A(x_k)$

$$Vol = \sum A(x_k) \cdot \Delta x$$

$$V = \int_a^b A(x) dx$$

= volume of a solid w/ cross-section Area $A(x)$ from $x=a$ to $x=b$

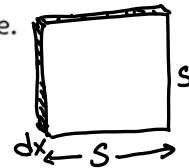
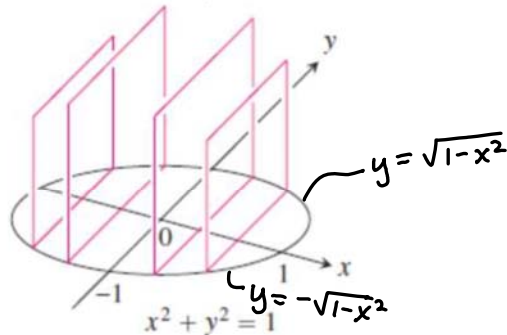
Find the volume of a sphere of radius R.





Find the volume of the solid created. The solids lay between planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross sections perpendicular to the x-axis between these planes run from the semi-circle $y = -\sqrt{1-x^2}$ to the semi-circle $y = \sqrt{1-x^2}$.

a) Cross sections are squares with bases in the xy-plane.



$$S = \sqrt{1-x^2} - (-\sqrt{1-x^2})$$

$$S = 2\sqrt{1-x^2}$$

$$A = S^2$$

$$A = (2\sqrt{1-x^2})^2$$

$$A = 4(1-x^2) = 4 - 4x^2$$

$$V = \int_{-1}^1 (4 - 4x^2) dx$$

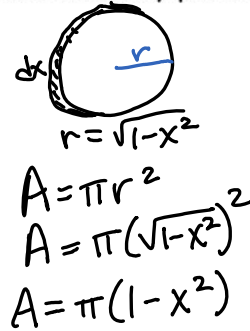
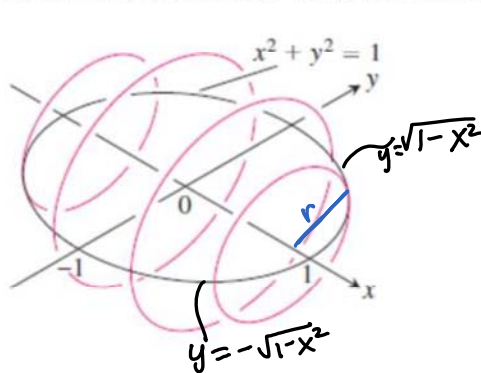
$$= 4x - \frac{4}{3}x^3 \Big|_{-1}^1$$

$$= 4(1) - \frac{4}{3}(1)^3 - (4(-1) - \frac{4}{3}(-1)^3)$$

$$= 4 - \frac{4}{3} + 4 - \frac{4}{3}$$

$$= \boxed{\frac{16}{3}}$$

b) Cross sections are circular disks with diameters in the xy-plane.



$$r = \sqrt{1-x^2}$$

$$A = \pi r^2$$

$$A = \pi(\sqrt{1-x^2})^2$$

$$A = \pi(1-x^2)$$

$$V = \int_{-1}^1 \pi(1-x^2) dx$$

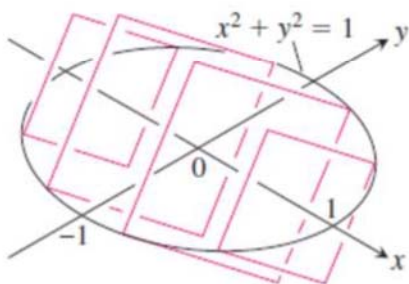
$$= \pi(x - \frac{1}{3}x^3) \Big|_{-1}^1$$

$$= \pi[(1 - \frac{1}{3}(1)^3) - ((-1) - \frac{1}{3}(-1)^3)]$$

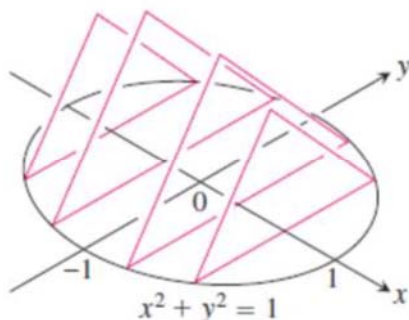
$$= \pi(\frac{2}{3} + \frac{2}{3})$$

$$= \boxed{\frac{4\pi}{3}}$$

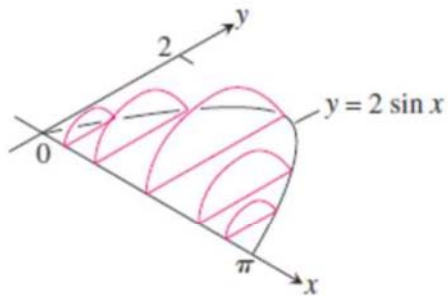
c) Cross sections are squares with diagonals in the xy-plane.



d) Cross sections are equilateral triangles with bases in the xy-plane.

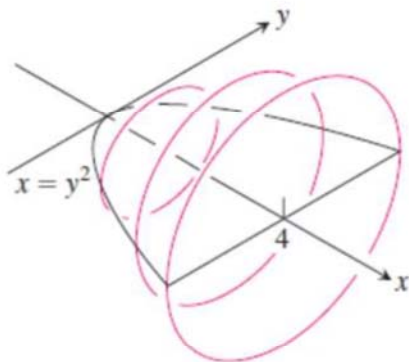


Find the volume of the solid created so its base is the shape of the region between the x-axis and one arch of the curve $y = 2 \sin x$. Each cross section cut perpendicular to the x-axis is a semi-circle whose diameter runs from the x-axis to the curve.



The solid lies between planes perpendicular to the x-axis at $x = 0$ and $x = 4$. The cross sections perpendicular to the x-axis between these planes run from $y = -\sqrt{x}$ to $y = \sqrt{x}$.

a) The cross sections are circular disks with diameters in the xy-plane.



b) The cross sections are squares with bases in the xy-plane.

