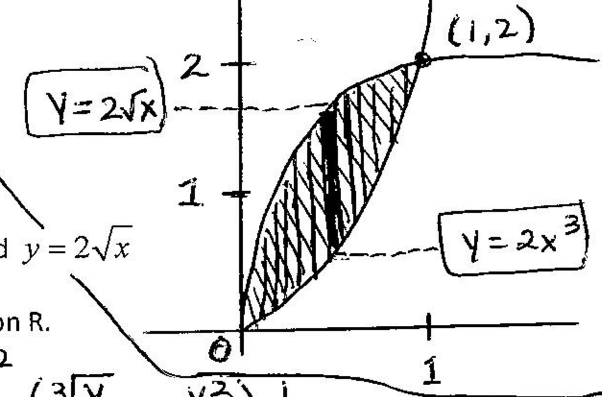
KEY

with respect to x (vertical slices)



AP Calculus AB

7.3 Area/Volume Review

Let R be the region enclosed by the curves $y = 2x^3$ and $y = 2\sqrt{x}$

1. Write an integral representing the area of region R.

vertical slices → $\int_0^1 \left(2\sqrt{x} - 2x^3\right)dx$ or $\int_0^2 \left(\sqrt[3]{\frac{y}{2}} - \frac{y^2}{4}\right)dy$ ← Horizontal slices

2. Write an integral representing the volume of a solid formed such that cross sections taken perpendicular to the x-axis are squares with one side running between the two curves.

vertical slices $\int_0^1 \left(2\sqrt{x} - 2x^3\right)^2 dx$ (Area $= (\text{side})^2$)

3. Write an integral representing the volume of a solid formed such that cross sections taken perpendicular to the y-axis are isosceles right triangles with one leg running between the two curves.

Horizontal slices $\int_0^2 \frac{1}{2}\left(\sqrt[3]{\frac{y}{2}} - \frac{y^2}{4}\right)^2 dy$ (Area $= \frac{1}{2}(\text{side})^2$)

4. Write an integral representing the volume of the solid formed by revolving R about the...

always slice perpendicular to axis of revolution

a) x-axis $\int_0^1 \left(\pi\left(2\sqrt{x} - 0\right)^2 - \pi\left(2x^3 - 0\right)^2\right)dx$

b) y-axis $\int_0^2 \left(\pi\left(\sqrt[3]{\frac{y}{2}} - 0\right)^2 - \pi\left(\frac{y^2}{4} - 0\right)^2\right)dy$

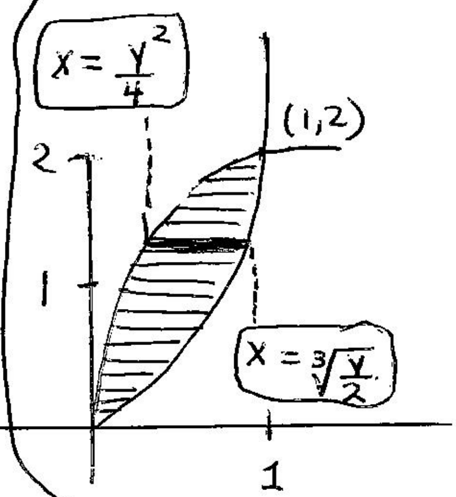
c) Line $x = -3$ $\int_0^2 \left(\pi\left(\sqrt[3]{\frac{y}{2}} - (-3)\right)^2 - \pi\left(\frac{y^2}{4} - (-3)\right)^2\right)dy$

d) Line $y = 2$ $\int_0^1 \left(\pi\left(2 - 2x^3\right)^2 - \pi\left(2 - 2\sqrt{x}\right)^2\right)dx$

* e) Line $x = 5$ $\int_0^2 \left(\pi\left(5 - \frac{y^2}{4}\right)^2 - \pi\left(5 - \sqrt[3]{\frac{y}{2}}\right)^2\right)dy$

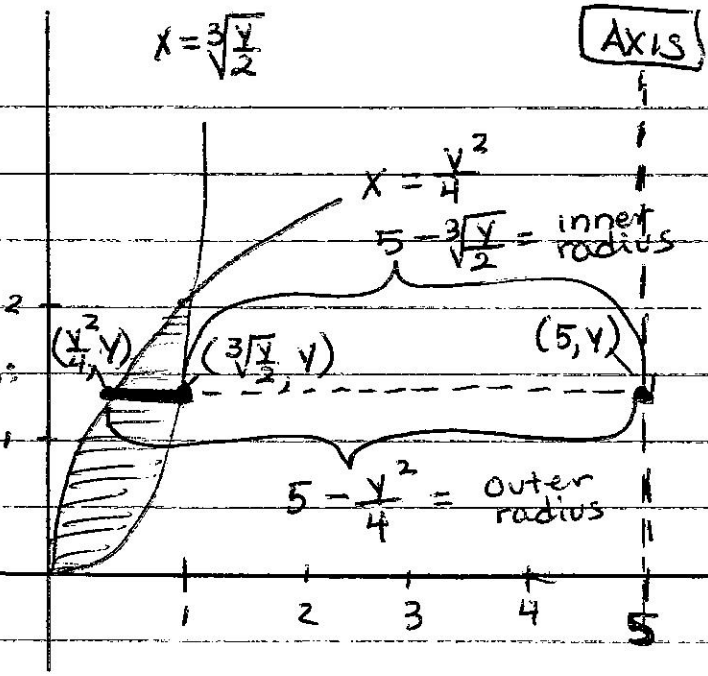
* f) Line $y = -1$ $\int_0^1 \left(\pi\left(2\sqrt{x} - (-1)\right)^2 - \pi\left(2x^3 - (-1)\right)^2\right)dx$

with respect to y (horizontal slices)



* see diagrams

e)



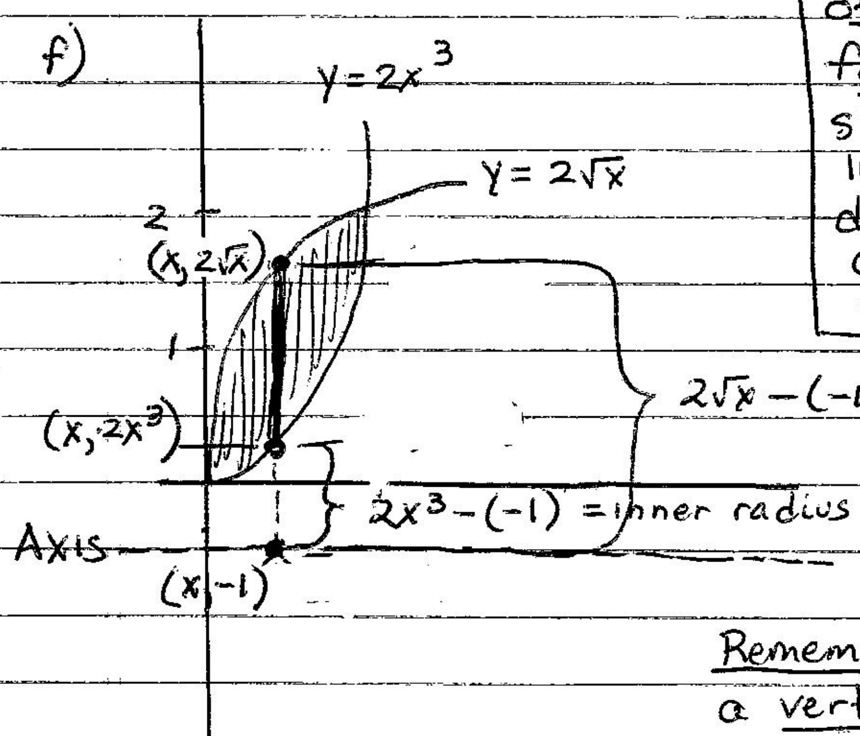
Limits of integration: Start slicing at $y=0$, stop at $y=2$

REMEMBER: THE Length of a horizontal distance is always measured "RIGHT MINUS LEFT"

WASHERS $\pi R^2 - \pi r^2$

The Outer radius (R) is always the distance from the axis of revolution to the farther edge of the shaded region. The Inner radius (r) is the distance from the axis to the nearer edge of the shaded region.

f)



$2\sqrt{x} - (-1)$ = outer radius

Remember: the length of a vertical distance is always measured "top minus bottom"

Limits of integration:
Start slicing at $x=0$,
Stop at $x=1$