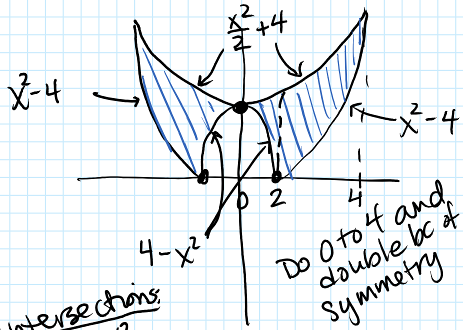
HW 8.2 p. 396 21, 22, 24, 26, 28, 32

Thursday, March 12, 2015 1:34 PM

(21) $y = |x^2 - 4|$ and $y = \left(\frac{x^2}{2}\right) + 4$



Intersections:

$\frac{x^2}{2} + 4 = x^2 - 4$

$x^2 + 8 = 2x^2 - 8$

$16 = x^2$

$\pm 4 = x$

$$A = 2\int_0^2 \left[\left(\frac{x^2}{2}+4\right) - (4 - x^2)\right]dx + 2\int_2^4 \left[\left(\frac{x^2}{2}+4\right) - (x^2-4)\right]dx$$

$$= 2\int_0^2 \frac{3}{2}x^2\,dx + 2\int_2^4 \left(-\frac{1}{2}x^2 + 8\right)dx$$

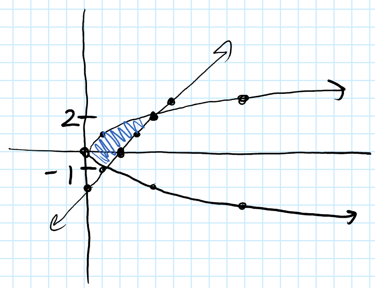
$$= 2\left(\frac{3}{2}\right)\left(\frac{1}{3}\right)x^3\Big|_0^2 + 2\left(\left(-\frac{1}{2}\right)\left(\frac{1}{3}\right)x^3 + 2(8x)\right)\Big|_2^4$$

$$= x^3\Big|_0^2 + \left(-\frac{1}{3}x^3 + 16x\right)\Big|_2^4$$

$$= 2^3 - 0^3 + -\frac{1}{3}(4)^3 + 16(4) - -\frac{1}{3}(2)^3 - 16(2)$$

$$= \boxed{21\tfrac{1}{3}}$$

(22) $x = y^2 \quad x = y + 2$



$$A = \int_{-1}^{2} [(y+2) - y^2]\,dy = \left[\frac{1}{2}y^2 + 2y - \frac{1}{3}y^3\right]_{-1}^{2}$$

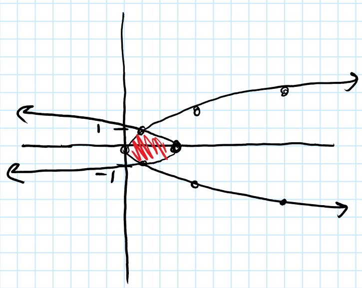
$$= \left(\frac{1}{2}(2)^2 + 2(2) - \frac{1}{3}(2)^3\right) - \left(\frac{1}{2}(-1)^2 + 2(-1) - \frac{1}{3}(-1)^3\right)$$

$$= \left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$$

$$= \boxed{4\tfrac{1}{2}}$$

(24) $x - y^2 = 0 \qquad x + 2y^2 = 3$

$x = y^2 \qquad x = -2y^2 + 3$



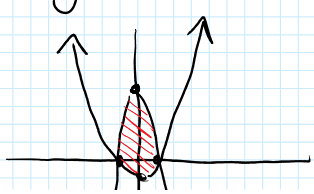
$$A = \int_{-1}^{1} \left((-2y^2+3) - y^2\right)dy = \int_{-1}^{1} (-3y^2 + 3)\,dy$$

$$= \left[-y^3 + 3y\right]_{-1}^{1} = \left(-(1)^3 + 3(1)\right) - \left(-(-1)^3 + 3(-1)\right)$$

$$= (2) - (-2) = \boxed{4}$$

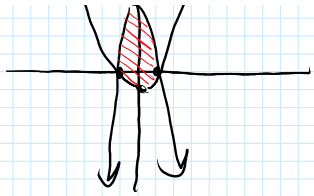
(26) $4x^2 + y = 4 \qquad x^4 - y = 1$

$y = -4x^2 + 4 \qquad y = x^4 - 1$



$$A = \int_{-1}^{1} [(-4x^2+4) - (x^4 - 1)]\,dx = \int_{-1}^{1} [-4x^2 - x^4 + 5]\,dx$$

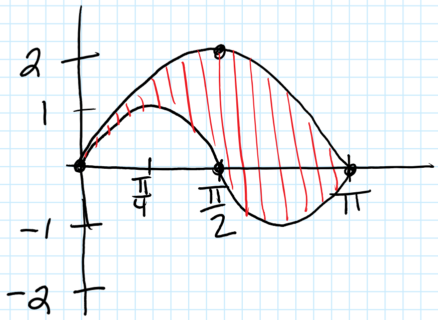
$$= \left(-\frac{4}{3}x^3 - \frac{1}{5}x^5 + 5x\right)\Big|_{-1}^{1} = \left(-\frac{4}{3}(1)^3 - \frac{1}{5}(1)^5 + 5(1)\right) - \left(-\frac{4}{3}(-1)^3 - \frac{1}{5}(-1)^5 + 5(-1)\right)$$



$= \left(-\frac{4}{3}x^3 - \frac{1}{5}x^5 + 5x\right)\Big|_{-1}^{1} = \left(-\frac{4}{3}(1)^3 - \frac{1}{5}(1)^5 + 5(1)\right) - \left(-\frac{4}{3}(-1)^3 - \frac{1}{5}(-1)^5 + 5(-1)\right)$

$= \boxed{6\frac{14}{15}}$

(28) $y = 2\sin x \quad y = \sin 2x \quad 0 \le x \le \pi$



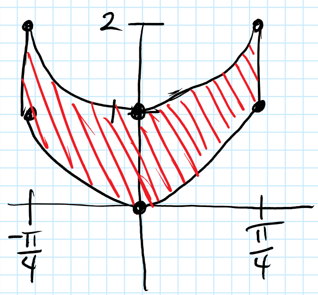
$A = \int_0^{\pi} [2\sin x - \sin 2x]\,dx = \left[-2\cos x + \frac{1}{2}\cos 2x\right]_0^{\pi}$

$= \left(-2\cos\pi + \frac{1}{2}\cos 2\pi\right) - \left(-2\cos 0 + \frac{1}{2}\cos 2(0)\right)$

$= \left(-2(-1) + \frac{1}{2}(1)\right) - \left(-2(1) + \frac{1}{2}(1)\right)$

$= \boxed{4}$

(32) $y = \sec^2 x \quad y = \tan^2 x \quad x = -\frac{\pi}{4},\ x = \frac{\pi}{4}$



Pythag Id

$A = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sec^2 x - \tan^2 x)\,dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 1\,dx = x\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$

$= \frac{\pi}{4} - \frac{-\pi}{4} = \boxed{\frac{\pi}{2}}$