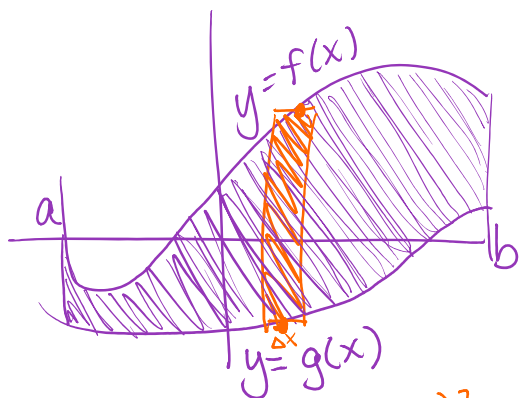


AP Calculus AB
7.2 Areas in the Plane

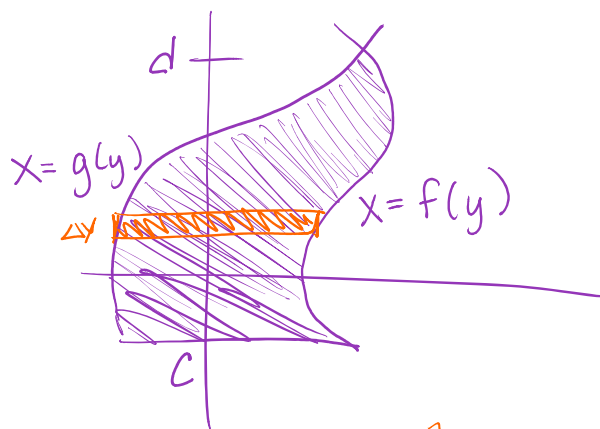
Name _____

Area Between Curves:



$$A = \sum [f(c_k) - g(c_k)] \Delta x$$

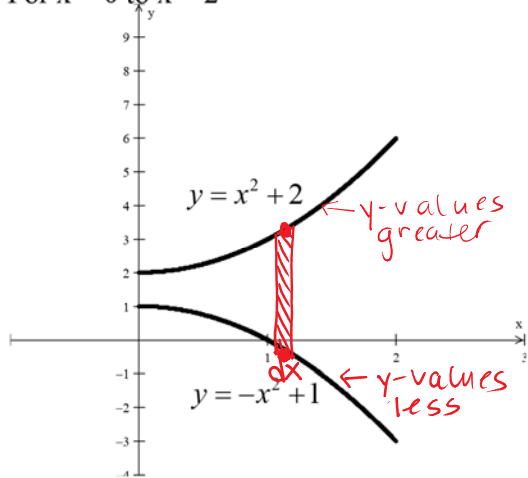
$$A = \int_a^b (f(x) - g(x)) dx$$



$$A = \sum [f(y) - g(y)] \Delta y$$

$$A = \int_c^d (f(y) - g(y)) dy$$

Find the area between the curves analytically.

1. For $x=0$ to $x=2$ 

$$A = \int_0^2 [(x^2+2) - (-x^2+1)] dx$$

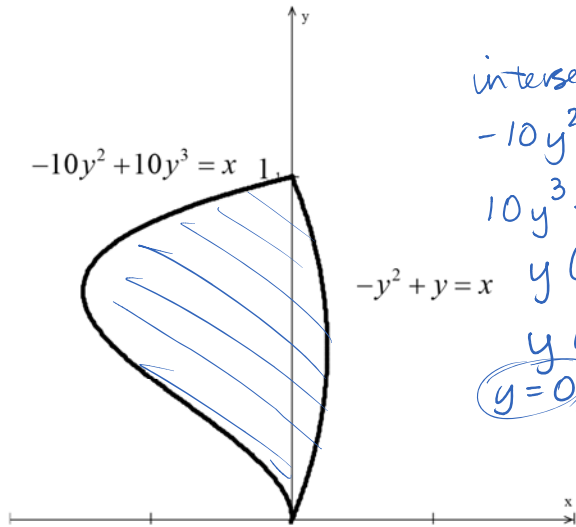
$$= \int_0^2 (2x^2 + 1) dx$$

$$= \left[\frac{2}{3}x^3 + x \right]_0^2$$

$$= \left(\frac{2}{3}(2)^3 + 2 \right) - \left(\frac{2}{3}(0)^3 + 0 \right)$$

$$= \frac{16}{3} + 2 = \frac{16}{3} + \frac{6}{3} = \boxed{\frac{22}{3}}$$

2.



intersection:

$$-10y^2 + 10y^3 = -y^2 + y$$

$$10y^3 - 9y^2 - y = 0$$

$$y(10y^2 - 9y - 1) = 0$$

$$y(10y + 1)(y - 1) = 0$$

$y=0, y=-\frac{1}{10}, 1$ enclosed area

$$A = \int_0^1 [(-y^2 + y) - (-10y^2 + 10y^3)] dy$$

$$= \int_0^1 (9y^2 + y - 10y^3) dy$$

$$= \left[3y^3 + \frac{1}{2}y^2 - \frac{10}{4}y^4 \right]_0^1 = (3(1)^3 + \frac{1}{2}(1)^2 - \frac{10}{4}(1)^4) - (0)$$

$$= 3 + \frac{1}{2} - \frac{5}{2} = \frac{6}{2} + \frac{1}{2} - \frac{5}{2} = \boxed{1}$$

3. Find the area enclosed by $y = 2x^2$ and $y = x^4 - 2x^2$ analytically.

Int:

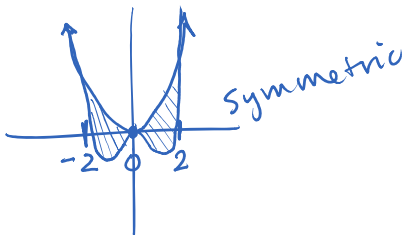
$$2x^2 = x^4 - 2x^2$$

$$0 = x^4 - 4x^2$$

$$0 = x^2(x^2 - 4)$$

$$0 = x^2(x+2)(x-2)$$

$$x = 0, -2, 2$$



$$A = 2 \int_0^2 (2x^2 - (x^4 - 2x^2)) dx$$

$$= 2 \int_0^2 (4x^2 - x^4) dx$$

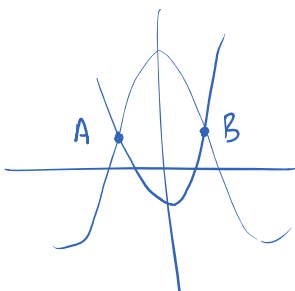
$$= 2 \left[\frac{4}{3}x^3 - \frac{1}{5}x^5 \right]_0^2 dx$$

$$= 2 \left(\frac{4}{3}(2)^3 - \frac{1}{5}(2)^5 \right) - (0)$$

$$= 2 \left(\frac{32}{3} - \frac{32}{5} \right)$$

$$= \boxed{\frac{128}{15}}$$

4. Find the area enclosed by $y = 2\cos x$ and $y = x^2 - 1$ (Calc OK)



$$A \approx -1.265$$

$$B \approx 1.265$$

$$A \approx \int_A^B (2\cos x - (x^2 - 1)) dx \approx \boxed{4.995}$$

J T U

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