8.1 Notes Day 1

Sunday, February 26, 2017 3:12 PM

$v \quad \int v \quad \int v \quad \int v \quad \int v$

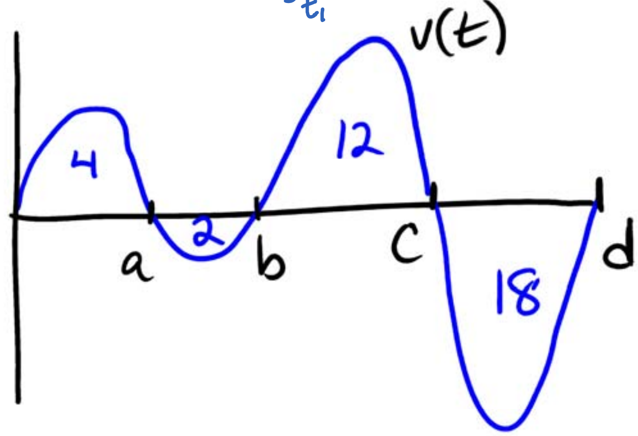
8.1 INTEGRAL AS NET CHANGE

Displacement **VS.** **Total Distance**

net change in position $= \int_{t_1}^{t_2} v(t)\,dt$

how far travelled: distance is positive $= \int_{t_1}^{t_2} |v(t)|\,dt$

A particle moves along the x-axis from time t = 0 to time *t = d*. Its initial position at *t = 0* is s(0) = 5. The graph shows the particle's velocity *v(t)*. The numbers are the areas of the enclosed regions.



Answer the following questions:

1) When is the particle moving to the…

Right? when $v(t) > 0$ $(0,a) \cup (b,c)$

Left? when $v(t) < 0$ $(a,b) \cup (c,d)$

When is it stopped? when $v(t) = 0$ $0, a, b, c, d$

2) What is the particle's **displacement** and **total distance** from…

$t = 0$ to $t = a$? $\int_0^a v(t)\,dt = \boxed{4}$ | $\int_0^a |v(t)|\,dt = 4$

$t = 0$ to $t = b$? $\int_0^b v(t)\,dt = 4 - 2 = \boxed{2}$ | $\int_0^b |v(t)|\,dt = \int_0^a v(t)\,dt - \int_a^b v(t)\,dt = 4 - (-2) = \boxed{6}$

$t = 0$ to $t = c$? $\int_0^c v(t)\,dt = 4 - 2 + 12 = \boxed{14}$ | $\int_0^c |v(t)|\,dt = \int_0^a v(t)\,dt - \int_a^b v(t)\,dt + \int_b^c v(t)\,dt = 4 - (-2) + 12 = \boxed{18}$

$t = 0$ to $t = d$? $\int_0^d v(t)\,dt = \int_0^c v(t)\,dt + (-18) = \boxed{-4}$ | $\int_0^d |v(t)|\,dt = \int_0^c |v(t)|\,dt - (-18) = \boxed{36}$

3) What is the particle's position at time…

a? $s(a) = s(0) + \int_0^a v(t)\,dt$

b? $s(b) = s(0) + \int_0^b v(t)\,dt$

c? $s(c) = s(0) + \int_0^c v(t)\,dt = 5 + 14 = \boxed{19}$

d? $s(d) = s(0) + \int_0^d v(t)\,dt = 5 + -4$

a? $s(a) = s(0) + \int_0^a v(t)\,dt$

$= 5 + 4 = \boxed{9}$

b? $s(b) = s(0) + \int_0^b v(t)\,dt$

$= 5 + 2 = \boxed{7}$

c? $s(c) = s(0) + \int_0^c v(t)\,dt$

$= 5 + 14 = \boxed{19}$

d? $s(d) = s(0) + \int_0 v(t)\,dt$

$= 5 + -4$

$= \boxed{1}$

For the following problems, the function v(t) is the velocity in cm/sec of a particle moving along the x-axis. Find the following:

a) When the particle is stopped, moving left, and moving right.
b) The displacement for the given time interval.
c) The particle's final position if s(0) = 5.
d) The total distance the particle traveled.

4) $v(t) = 4\cos 2t, \quad 0 \le t \le \pi$

~~crit pts~~

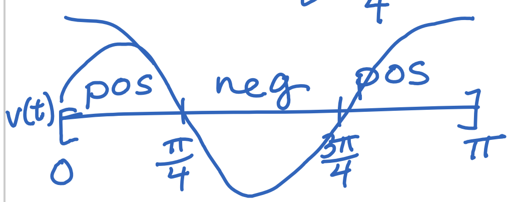
$v(t) = 4\cos 2t = 0$

$\boxed{\cos 2t = 0}$

$\cos^{-1}(0) = 2t$

$2t = \frac{\pi}{2} \qquad 2t = \frac{3\pi}{2}$

$t = \frac{\pi}{4} \qquad t = \frac{3\pi}{4}$



ⓐ Stopped: $v(t) = 0 : t = \frac{\pi}{4}, \frac{3\pi}{4}$

moving right: $v(t) > 0: (0, \frac{\pi}{4}) \cup (\frac{3\pi}{4}, \pi)$

moving left: $v(t) < 0: (\frac{\pi}{4}, \frac{3\pi}{4})$

ⓑ displacement $= \int_0^{\pi} 4\cos 2t\,dt$

$u = 2t$
$du = 2dt$
$2du = 4dt$
$u(0) = 0$
$u(\pi) = 2\pi$

$= \int_0^{2\pi} 2\cos u\,du$

$= 2\sin u \Big|_0^{2\pi} = 2\sin(2\pi) - 2\sin 0$
$= \boxed{0}$

ⓒ $s(\pi) = s(0) + \int_0^{\pi} v(t)\,dt$

$s(\pi) = 5 + 0 = \boxed{5}$

ⓓ $\int_0^{\pi} |v(t)|\,dt =$

$\int_0^{\frac{\pi}{4}} 4\cos 2t\,dt - \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 4\cos 2t\,dt + \int_{\frac{3\pi}{4}}^{\pi} 4\cos 2t\,dt$

$= 2\sin 2t \Big|_0^{\frac{\pi}{4}} - 2\sin 2t \Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}} + 2\sin 2t \Big|_{\frac{3\pi}{4}}^{\pi}$

$= (2\sin\frac{\pi}{2} - 2\sin 0) - (2\sin\frac{3\pi}{2} - 2\sin\frac{\pi}{2}) + (2\sin 2\pi - 2\sin\frac{3\pi}{2})$

$= (2 - 0) - (-2 - 2) + (0 - (-2))$

$= 2 + 4 + 2 = \boxed{8}$

5) $v(t) = 49 - 9.8t, \quad 0 \le t \le 8$