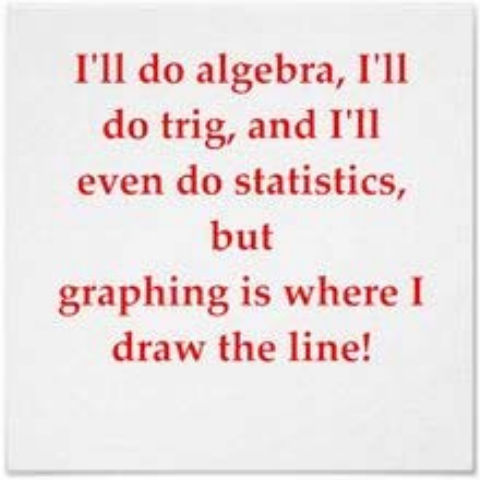
Thursday, February 16, 2017

- 7.1 Opener - handout
- Slope Field Green Sheet Check-in
- 7.4 - Separable Differential Equations



(a) III
(b) II
(c) IV
(d) I
(e) V

$y = xe^x$

$\frac{dy}{dx} = e^x + xe^x$

$\frac{d^2y}{dx^2} = e^x + e^x + xe^x$

7.4 Separable Differential Equations

$\frac{dy}{dx} = f(y) \cdot g(x)$ Separate + Integrate

$\int \frac{1}{f(y)} \cdot dy = \int g(x) \cdot dx$ Antiderivatwie (only one Constant)

(1) Solve for y if $\frac{dy}{dx} = x^2 y$ Separate Variables

$\int \frac{1}{y} dy = \int x^2 dx$ Integrate Both Sides

$\ln|y| = \frac{1}{3}x^3 + C$ Antiderivative of Both Sides (one C)

$|y| = e^{\frac{1}{3}x^3 + C}$

Solve for y

$y = \pm e^{\frac{1}{3}x^3 + C}$

$y = \pm e^{\frac{1}{3}x^3} \cdot e^C$ e^C is a Constant

$y = \pm C e^{\frac{1}{3}x^3}$

$y = \pm Ce^{\frac{1}{3}x}$

(2) $\frac{dy}{dx} = \frac{x}{y}$ and $y = 2$ when $x = 1$

find particular solution y

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\int y \cdot dy = \int x\,dx$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$$

Find C! →

$$\frac{1}{2}(2)^2 = \frac{1}{2}(1)^2 + C$$

$$2 = \frac{1}{2} + C$$

$$\frac{3}{2} = C$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + \frac{3}{2}$$

$$\sqrt{y^2} = \sqrt{x^2 + 3}$$

$$y = \pm\sqrt{x^2 + 3}$$

Solve for y

$(1, 2)$ on positive side of solution

$$\boxed{y = \sqrt{x^2 + 3}}$$

(3) $\frac{dy}{dx} = (y - 3)(x + 4)$ and $y = 5$ when $x = 0$

$$\int \frac{1}{y-3}\,dy = \int (x+4)\,dx$$

$$\ln|y - 3| = \frac{1}{2}x^2 + 4x + C$$

$$\ln|5 - 3| = \frac{1}{2}(0)^2 + 4(0) + C \quad \leftarrow \text{find } C$$

$$\ln 2 = C$$

$$\ln|y - 3| = \frac{1}{2}x^2 + 4x + \ln 2$$

$$|y - 3| = e^{\frac{1}{2}x^2 + 4x + \ln 2}$$

$$y - 3 = \pm e^{\frac{1}{2}x^2 + 4x + \ln 2}$$

$e^{\ln 2} = 2$

$$y-3=\pm e^{\frac{1}{2}x^2+4x}$$

$$y=\pm e^{\frac{1}{2}x^2+4x}\cdot e^{\ln 2}+3$$

$$y=\pm 2e^{\frac{1}{2}x^2+4x}+3$$

$e^{\ln 2}=2$

Restrict $(0,5)$ on positive side

$$y=2e^{\frac{1}{2}x^2+4x}+3$$