

② $\frac{dy}{dx} = -\frac{x}{y}$ $y=3$ when $x=4$
 $\int y dy = -\int x dx$
 $\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C_1$
 $(3)^2 = -(4)^2 + C_2$
 $9 = -16 + C$
 $25 = C$

pos only (4,3)

$y^2 = -x^2 + 25$
 $y = \sqrt{25 - x^2}$
 for $-5 \leq x \leq 5$

③ $\frac{dy}{dx} = \frac{y}{x}$ $y=2$ when $x=2$
 $\int \frac{1}{y} dy = \int \frac{1}{x} dx$ VA $x=0$ (2,2) pos side
 $\ln|y| = \ln|x| + C \rightarrow \ln|y| = \ln|x|$
 $\ln 2 = \ln 2 + C$
 $0 = C$
 $\ln y = \ln x$
 $y = x$
 for $x > 0$

④ $\frac{dy}{dx} = 2xy$ $y=3$ $x=0$
 $\int \frac{1}{y} dy = \int 2x dx$ cont. (0,3) pos
 $\ln|y| = x^2 + C \rightarrow \ln|y| = x^2 + \ln 3$
 $\ln 3 = 0 + C$
 $\ln 3 = C$
 $e^{x^2 + \ln 3} = |y|$
 $|y| = e^{x^2} \cdot e^{\ln 3}$
 $y = 3e^{x^2}$
 for all reals

⑤ $\frac{dy}{dx} = (y+5)(x+2)$ $y=1$ $x=0$
 $\int \frac{1}{y+5} dy = \int (x+2) dx$ cont (0,1) pos
 $\ln|y+5| = \frac{1}{2}x^2 + 2x + C$
 $\ln 6 = C$
 $\ln y+5 = \frac{1}{2}x^2 + 2x + \ln 6$
 $y+5 = e^{\frac{1}{2}x^2 + 2x + \ln 6}$
 $y+5 = 6e^{\frac{1}{2}x^2 + 2x}$
 $y = 6e^{\frac{1}{2}x^2 + 2x} - 5$
 all reals

⑥ $\frac{dy}{dx} = \cos^2 y$ $y=0$ when $x=0$
 $\frac{dy}{dx} = \frac{1}{\sec^2 y}$ continuous
 $\int \sec^2 y dy = \int 1 dx$
 $\tan y = x + C$
 $\tan 0 = 0 + C$
 $0 = C$
 $y = \tan^{-1} x$
 for all reals

⑦ $\frac{dy}{dx} = (\cos x)e^{y+\sin x}$ $y=0$ $x=0$
 $\frac{dy}{dx} = (\cos x)e^y e^{\sin x}$ continuous
 $\int e^{-y} dy = \int \cos x e^{\sin x} dx$
 $-e^{-y} = e^{\sin x} + C \rightarrow e^{-y} = -e^{\sin x} + 2$
 $-e^0 = e^{\sin 0} + C$
 $-1 = 1 + C$
 $-2 = C$
 $-y = \ln(-e^{\sin x} + 2)$
 $y = -\ln(2 - e^{\sin x})$
 for all reals

⑧ $\frac{dy}{dx} = e^{x-y}$ $y=2$ when $x=0$
 $\frac{dy}{dx} = \frac{e^x}{e^y}$ cont.

$$\int e^y dy = \int e^x dx$$

$$e^y = e^x + C$$

$$e^2 = e^0 + C$$

$$e^2 - 1 = C$$

$$\rightarrow e^y = e^x + e^2 - 1$$

$$y = \ln(e^x + e^2 - 1)$$

for all reals