7.4 Exponential Growth and Decay

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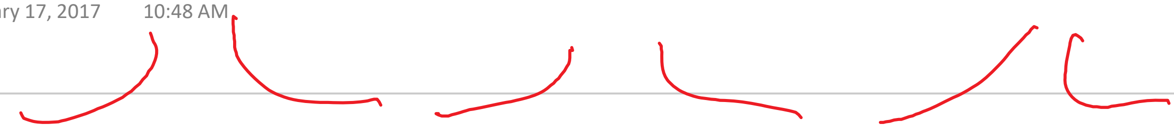

7.4 Exponential Growth and Decay Notes **Name:** ____________________

In many applications, the rate of change of a variable y is proportional to the value of y. If y is a function of time t, we can express this statement as

$$\frac{dy}{dt} = ky$$

$k =$ growth/decay constant

Example 1: Find the solution to this differential equation given the initial condition that y = y_0 when t = 0. (This is the derivation of an exponential function AND you want to know how to do it yourself!)

$$\frac{dy}{dt} = ky$$

$$\int \frac{1}{y}\,dy = \int k\,dt$$

$$\ln|y| = kt + C$$

$$|y| = e^{kt+C}$$

$$y = \pm e^{kt+C}$$

$$y = \pm e^{C} \cdot e^{kt}$$

$$\boxed{y = y_0 e^{kt}}$$

make $y_0 = \pm e^{C} =$ constant because when $t = 0$ $y = \pm e^{C}$ (initial amount)

Exponential Growth and Decay Model

If y changes at a rate proportional to the amount present ($\frac{dy}{dt} = ky$) and $y = y_0$ when $t = 0$, then

$$y = y_0 e^{kt}$$

where k is the **proportional constant**.

Exponential **growth** occurs when $k > 0$, and exponential **decay** occurs when $k < 0$.

Example 2: The rate of change of y is proportional to y. When t = 0, y = 2. When t = 2, y = 4. What is the value of y when t = 3?

$\frac{dy}{dt} = ky$ $\quad y_0 = 2$

$$y = y_0 e^{kt}$$

plug in (2, 4) for (t, y) and $y_0 = 2$, solve for k

$$4 = 2e^{k(2)}$$

$$2 = e^{2k}$$

$$\ln 2 = 2k$$

$$k \approx .347$$

exp growth model $\boxed{y = 2e^{.347t}}$

$$y(3) = 2e^{.347(3)} \approx \boxed{5.657}$$

Newton's Law of Cooling states that the rate of change in the temperature of an object is proportional to the difference between the object's temperature and the temperature in the surrounding medium. If the temperature is a function of time, then we can express this statement as:

$$T - T_s = (T_0 - T_s)e^{-kt}$$

T = Temp of object @ time t
T_s = surrounding temp
T_0 = initial temp of object

* works in °F, C

Example 3: A hard-boiled egg at 98˚C is put in a pan under running 18˚C water to cool. After 5 minutes, the egg's temperature is found to be 38˚C. How much longer will it take the egg to reach 20˚C?

$t = 5$ min

$T_s = 18°$
$T = 38°$
$T_0 = 98°$

$$38 - 18 = (98 - 18)e^{-k(5)}$$
$$20 = 80e^{-k(5)}$$
$$\frac{1}{4} = e^{-k(5)}$$
$$\ln\frac{1}{4} = -k(5)$$
$$k \doteq .277$$

$$T - 18 = (98 - 18)e^{-.277t}$$

$T = 20°C$ what is $t = ?$

$$20 - 18 = 80e^{-.277t}$$
$$2 = 80e^{-.277t}$$
$$t \doteq 13.305$$
$$-\ 5 \text{ min}$$

8.305 minutes longer