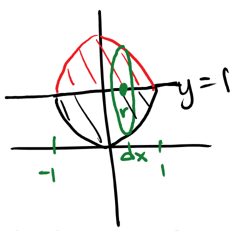
AP Calculus AB

7.3 Practice

1. Find the volume of the solid generated by revolving the region bounded by the parabola $y = x^2$ and the line $y = 1$ about:

a) the line $y = 1$



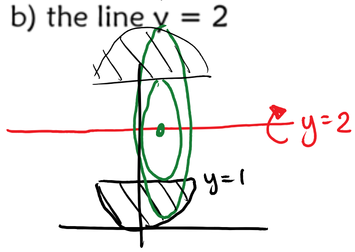
$r = 1 - x^2$

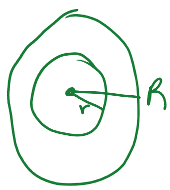
$A = \pi(1-x^2)^2$

$= \pi(1 - 2x^2 + x^4)$

$V = \int_{-1}^{1} \pi(1-2x^2+x^4)\,dx = \left[\pi\left(x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right)\right]_{-1}^{1} = \boxed{\frac{16\pi}{15}}$

b) the line $y = 2$





$R = 2 - x^2$

$r = 2 - 1 = 1$

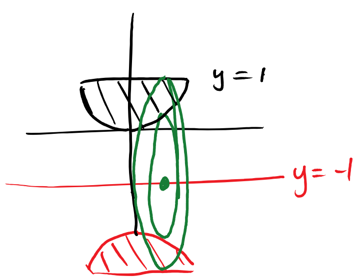
$A = \pi(2-x^2)^2 - \pi(1)^2$

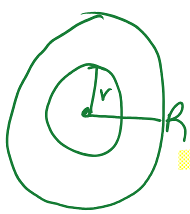
$= \pi(4 - 4x^2 + x^4) - \pi$

$= \pi(3 - 4x^2 + x^4)$

$V = \int_{-1}^{1} \pi(3 - 4x^2 + x^4)\,dx = \pi\left[3x - \frac{4}{3}x^3 + \frac{1}{5}x^5\right]_{-1}^{1} = \boxed{\frac{56\pi}{15}}$

c) the line $y = -1$





$R = -1 - 1 = -2$ (or $R = 2$)

$r = -1 - x^2$ (or $r = 1 + x^2$)

$A = \pi(-2)^2 - \pi(-1-x^2)^2$

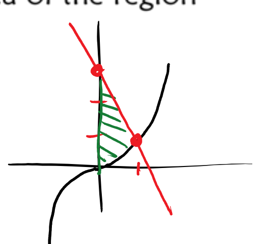
$= 4\pi - \pi(1 + 2x^2 + x^4)$

$= \pi(3 - 2x^2 - x^4)$

$V = \int_{-1}^{1} \pi(3 - 2x^2 - x^4)\,dx = \left[\pi\left(3x - \frac{2}{3}x^3 - \frac{1}{5}x^5\right)\right]_{-1}^{1} = \boxed{\frac{64\pi}{15}}$

2. A region is bounded by the curves $y = x^3$, $y = -2x + 3$, and $x = 0$.

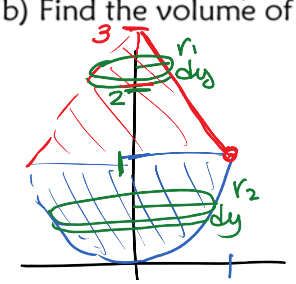
a) Find the area of the region



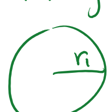
$A = \int_0^1 (-2x + 3 - x^3)\,dx = \left[-x^2 + 3x - \frac{1}{4}x^4\right]_0^1$

$= \boxed{\frac{7}{4}}$

b) Find the volume of the solid generated when the region is rotated about the y-axis.



2 integrals



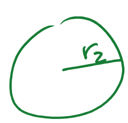
$y = -2x + 3$

$y - 3 = -2x$

$x = -\frac{1}{2}y + \frac{3}{2}$

$r = -\frac{1}{2}y + \frac{3}{2}$

$A_1 = \pi\left(-\frac{1}{2}y + \frac{3}{2}\right)^2$

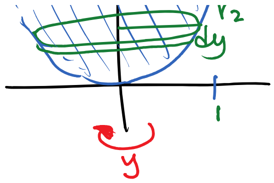


$y = x^3$

$x = \sqrt[3]{y}$

$r_2 = \sqrt[3]{y}$

$A_2 = \pi\left(\sqrt[3]{y}\right)^2 = \pi y^{\frac{2}{3}}$



$r = -\frac{1}{2}y + \frac{3}{2}$

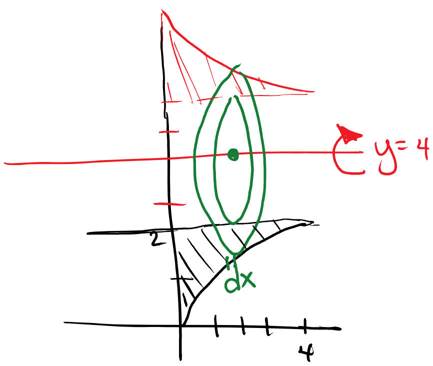
$A_2 = \pi(\sqrt[3]{y})^2 = \pi y^{2/3}$

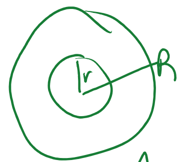
$A_1 = \pi\left(-\frac{1}{2}y + \frac{3}{2}\right)^2$

$$V = \int_1^3 \pi\left(-\frac{1}{2}y + \frac{3}{2}\right)^2 dy + \int_0^1 \pi y^{\frac{2}{3}}\, dy \approx \boxed{3.979}$$

3. Find the volume of the solid generated by revolving the given region around the line y = 4.

$y = 2, \quad y = \sqrt{x}, \quad y - axis$





$r = 4 - 2 = 2$

$R = 4 - \sqrt{x}$

$$A = \pi(4-\sqrt{x})^2 - \pi(2)^2$$
$$= \pi(16 - 8\sqrt{x} + x) - 4\pi$$
$$= \pi(12 - 8\sqrt{x} + x)$$

$$V = \int_0^4 \pi(12 - 8\sqrt{x} + x)\,dx = \left[\pi\left(12x - 8\cdot\frac{2}{3}x^{\frac{3}{2}} + \frac{1}{2}x^2\right)\right]_0^4$$

$$= \pi\left[12x - \frac{16}{3}x^{\frac{3}{2}} + \frac{1}{2}x^2\right]_0^4 = 13\frac{1}{3}\pi = \boxed{\frac{40\pi}{3}}$$