7.2 Multiplication Notes

Tuesday, October 18, 2016 7:30 AM

Precalculus 7.2 Notes Sheet: Matrix Multiplication

Example: Sara, Tara, and Kara want to go out to breakfast together. They decide to visit either Dunkin Donuts or Starbucks. On the left is the cost matrix for the two stores. It tells the price per item for several menu items at each store. On the right is the item matrix, which tells what the girls want to order.

	Coffee	Muffin	Scone
Dunkin	2.00	2.50	1.50
Starbucks	3.00	2.25	2.00

	Sara	Tara	Kara
Coffee	1	0	1
Muffin	2	1	0
Scone	0	1	3

Cost matrix dimensions: 2x3 **Item matrix dimensions:** 3x3

Find the cost of Sara's order at Dunkin.

2.00(1)+2.50(2) = $7.00

Find the cost of Tara's order at Dunkin.

Find the cost of Sara's order at Starbucks.

Find the cost of Tara's order at Starbucks.

2.25(1)+ 2.00(1) = 4.25

Find the cost of Kara's order at Dunkin.

Find the cost of Kara's order at Starbucks.

We can simplify what we just did by writing this as an equation with matrices:

$$\begin{matrix} & C & M & S \\ \text{Dunk} \\ \text{Star} \end{matrix}\begin{bmatrix} 2.00 & 2.50 & 1.50 \\ & & \end{bmatrix} \quad \begin{matrix} & \text{Sara} & \text{Tara} & \text{Kara} \\ C \\ M \\ S \end{matrix}\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \quad \begin{matrix} & \text{Sara} & \text{Tara} & \text{Kara} \\ \text{Dunk} \\ \text{Star} \end{matrix}\begin{bmatrix} 7.00 & 4.00 & 6.50 \\ 7.50 & 4.25 & 9.00 \end{bmatrix}$$

2x3 3x3 2x3

Cost matrix dimensions · Item matrix dimensions = Total cost dimensions

Multiplication Summary:

To multiply, Columns of Matrix 1 must = Rows of Matrix 2

Product will be Rows of Matrix 1 by Columns of Matrix 2

Multiply Rows of Matrix 1 by Columns of Matrix 2

Product will be Rows of Matrix 1 / Columns of Matrix 2

Multiply Rows of Matrix 1 by Columns of Matrix 2
(multiply each element and add)

Determine whether each matrix product is defined. If so, state the dimensions of the product.

1. $A_{3\times 5} \cdot M_{5\times 8}$ 2. $M_{2\times 1} \cdot A_{1\times 6}$ 3. $M_{3\times 2} \cdot A_{3\times 2}$

Step-By-Step Matrix Multiplication

- Step 1: Determine whether each product is defined and the dimensions of the product.
- Step 2: Follow the process outlined below to multiply the matrices together.

1. $\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 7 \\ 6 & 0 & -5 \end{bmatrix} = \begin{bmatrix} 30 & -4 & -6 \\ 3 & -6 & 26 \end{bmatrix}$

$2\times 2 \quad 2\times 3 \qquad 2\times 3$

$a_{11} = 2(3)+4(6)=30 \quad a_{12} = 2(-2)+4(0)=-4 \quad a_{13} = 2(7)+4(-5)=-6$

$a_{21} = 3(3)+(-1)(6)=3 \quad a_{22} = 3(-2)+(-1)(0)=-6 \quad a_{23} = 3(7)+(-1)(-5)=26$

2. $\begin{bmatrix} -3 & 0 \\ 2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ 7 & -1 \end{bmatrix} = \begin{bmatrix} -6 & -12 \\ 39 & 3 \end{bmatrix}$

3. $\begin{bmatrix} 4 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} = [2]$

$1\times 3 \cdot 3\times 1 = 1\times 1$

$4(1)+0(3)+2(-1)$

4. $\begin{bmatrix} 2 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 20 & 44 \end{bmatrix}$

$1\times 2 \quad 2\times 2 \qquad 1\times 2$

$2(1)+6(3)=20$

$2(4)+6(6)=44$

5. $\begin{bmatrix} -2 & 3 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 14 & -10 \\ 4 & -2 \end{bmatrix}$

6. $\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 8 \\ -5 & 10 \\ -6 & 12 \end{bmatrix}$

7. $\begin{bmatrix} 1 & 5 \\ 2 & 6 \\ 4 & 7 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 5 \end{bmatrix} = \begin{bmatrix} 32 \\ 44 \\ 63 \end{bmatrix}$

8. **Solve for a.** $\begin{bmatrix} 4 & 8 & a \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix} = [3]$