HW 7.2 p. 338 34, 39, 41, 47, 51, 52, 54, 57-59

Monday, February 16, 2015 1:52 PM

(34) $\int \tan^7 \frac{x}{2} \sec^2 \frac{x}{2}$

$u = \tan\left(\frac{x}{2}\right)$

$\frac{du}{dx} = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$

$du = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx$

$2du = \sec^2\left(\frac{x}{2}\right)dx$

$= 2\int u^7 du$

$= 2\left(\frac{1}{8}u^8\right) + C$

$= \boxed{\frac{1}{4}\left(\tan^8 \frac{x}{2}\right) + C}$

(39) $\int \frac{dx}{x \ln x}$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x}dx$

$= \int \frac{du}{u}$

$= \ln|u| + C$

$= \boxed{\ln(\ln x) + C}$

always +

(41) $\int \frac{x\,dx}{x^2+1}$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int \frac{du}{u}$

$= \frac{1}{2}\ln|u| + C$

$= \boxed{\frac{1}{2}\ln(x^2+1) + C}$

always +

(47) $\int \sin^3 2x\,dx$, $\sin^2 2x = 1 - \cos^2 2x$

$= \int (\sin^2 2x)(\sin 2x)dx$

$= \int (1-\cos^2 2x)(\sin 2x)dx$

$u = \cos 2x$

$\frac{du}{dx} = -\sin 2x(2)$

$du = -2\sin 2x\,dx$

$-\frac{1}{2}du = \sin 2x\,dx$

$= -\frac{1}{2}\int (1-u^2)du$

$= -\frac{1}{2}\int 1\,du + \frac{1}{2}\int u^2 du$

$= -\frac{1}{2}u + \frac{1}{6}u^3 + C$

$= \boxed{-\frac{1}{2}\cos 2x + \frac{1}{6}\cos^3 2x + C}$

(51) $\int \tan^4 x\,dx$, $\tan^2 x = \sec^2 x - 1$

$= \int \tan^2 x \cdot \tan^2 x\,dx$

$= \int (\sec^2 x - 1)\tan^2 x\,dx$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$du = \sec^2 x\,dx$

$= \int (\sec^2 x \tan^2 x - \tan^2 x)dx$

$= \int \sec^2 x \tan^2 x\,dx - \int \tan^2 x\,dx$

$= \int u^2 du - \int (\sec^2 x - 1)dx$

$= \frac{1}{3}u^3 - \tan x + x + C$

$\boxed{= \frac{1}{3}\tan^3 x - \tan x + x + C}$

(52) $\int (\cos^4 x - \sin^4 x)\,dx$ $\quad \cos 2x = \cos^2 x - \sin^2 x$

$\int (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)\,dx$

$\int \cos 2x\,(1)\,dx$

$\boxed{= \frac{1}{2}\sin 2x + C}$

(54) $\int_0^1 r\sqrt{1-r^2}\,dr$ $\quad u = 1 - r^2 \quad u(0) = 1$

$\frac{du}{dr} = -2r \quad u(1) = 0$

$= -\frac{1}{2}\int_1^0 \sqrt{u}\,du =$ $\quad du = -2r\,dr$

$-\frac{1}{2}du = r\,dr$

$= -\frac{1}{2}\left(\frac{2}{3}u^{\frac{3}{2}}\right)\Big|_1^0$

$= -\frac{1}{3}(0)^{\frac{3}{2}} - -\frac{1}{3}(1)^{\frac{3}{2}} = \boxed{\frac{1}{3}}$

(57) $\int_0^1 \frac{10\sqrt{\theta}}{(1+\theta^{\frac{3}{2}})^2}\,d\theta$ $\quad u = 1 + \theta^{\frac{3}{2}} \quad u(0) = 1$

$\frac{du}{d\theta} = \frac{3}{2}\theta^{\frac{1}{2}} \quad u(1) = 2$

$\left[du = \frac{3}{2}\theta^{\frac{1}{2}}d\theta\right]\frac{20}{3}$

$= \frac{20}{3}\int_1^2 \frac{du}{u^2}$ $\quad \frac{20}{3}du = 10\theta^{\frac{1}{2}}d\theta$

$= \frac{20}{3}(-1)u^{-1}\Big|_1^2$

$= \frac{-20}{3u}\Big|_1^2 = \frac{-20}{3(2)} - \frac{-20}{3(1)} = \frac{-20}{6} + \frac{20}{3} = \boxed{\frac{10}{3}}$

(58) $\int_{-\pi}^{\pi} \frac{\cos x}{\sqrt{4 + 3\sin x}}\,dx$ $\quad u = 4 + 3\sin x \quad u(-\pi) = 4$

$\frac{du}{dx} = 3\cos x \quad u(\pi) = 4$

$= \frac{1}{3}\int^4 \frac{du}{}$ $\quad du = 3\cos x\,dx$

$\frac{du}{dx} = 3\cos x$

$du = 3\cos x\,dx$

$\frac{1}{3}du = \cos x\,dx$

$u(1) = 4$

$$= \frac{1}{3}\int_4^4 \frac{du}{\sqrt{u}}$$

$$= \boxed{0}$$

(59) $$\int_0^1 \sqrt{t^5 + 2t}\,(5t^4 + 2)\,dt$$

$u = t^5 + 2t$

$\frac{du}{dt} = 5t^4 + 2$

$du = (5t^4 + 2)dt$

$u(0) = 0$

$u(1) = 3$

$$= \int_0^3 \sqrt{u}\,du = \int_0^3 u^{\frac{1}{2}}\,du$$

$$= \frac{2}{3}u^{\frac{3}{2}}\Big|_0^3 = \frac{2}{3}(3)^{\frac{3}{2}} - \frac{2}{3}(0)^{3/2} = \frac{2\sqrt{27}}{3} = \boxed{2\sqrt{3}}$$