

(22)  $\int \frac{9r^2 dr}{\sqrt{1-r^3}}$

$u = 1-r^3$   
 $\frac{du}{dr} = -3r^2$   
 $-3[du = -3r^2 dr]$   
 $-3du = 9r^2 dr$

$= \int \frac{-3du}{\sqrt{u}}$   
 $= \int -3u^{-\frac{1}{2}} du = -3(\frac{2}{1-\frac{1}{2}})u^{\frac{1}{2}} + C$   
 $= -6\sqrt{1-r^3} + C$

Check  $\frac{d}{dx}(-6\sqrt{1-r^3} + C) = -6(\frac{1}{2}(1-r^3)^{-\frac{1}{2}}(-3r^2)) + 0$   
 $= \frac{9r^2}{\sqrt{1-r^3}} \checkmark$

(23)  $\int (1-\cos \frac{t}{2})^2 \sin \frac{t}{2} dt$

$u = 1-\cos \frac{t}{2}$   
 $\frac{du}{dt} = \sin \frac{t}{2} \cdot \frac{1}{2}$   
 $du = \frac{1}{2} \sin \frac{t}{2} dt$   
 $2du = \sin \frac{t}{2} dt$

$= \int 2u^2 du$   
 $= 2(\frac{1}{3})u^3 + C$   
 $= \frac{2}{3}(1-\cos \frac{t}{2})^3 + C$

check:  $\frac{d}{dx}(\frac{2}{3}(1-\cos \frac{t}{2})^3 + C)$   
 $= \frac{2}{3}(3)(1-\cos \frac{t}{2})^2(\sin \frac{t}{2})(\frac{1}{2}) + 0$   
 $= (1-\cos \frac{t}{2})^2(\sin \frac{t}{2}) \checkmark$

(25)  $\int \frac{dx}{(1-x)^2}$

$u = 1-x$   
 $\frac{du}{dx} = -1$   
 $du = -dx$   
 $-du = dx$

$= \int \frac{-du}{u^2}$   
 $= \int -u^{-2} du$   
 $= +\frac{1}{u} + C$   
 $= \frac{1}{1-x} + C$

(27)  $\int \sqrt{\tan x} \sec^2 x dx$

$u = \tan x$   
 $\frac{du}{dx} = \sec^2 x$   
 $du = \sec^2 x dx$

$= \int \sqrt{u} du$   
 $= \frac{2}{3} u^{\frac{3}{2}} + C$   
 $= \frac{2}{3}(\tan x)^{\frac{3}{2}} + C$

(30)  $\int 3(\sin x)^{-2} dx$

$= \int 3 \csc^2 x dx$   
 $= -3 \cot x + C$

(33)  $\int \frac{\ln^6 x}{x} dx$

$u = \ln x$   
 $\frac{du}{dx} = \frac{1}{x}$   
 $du = \frac{1}{x} dx$

$= \int u^6 du$   
 $= \frac{1}{7} u^7 + C$   
 $= \frac{1}{7} \ln^7 x + C$

(35)  $\int 8^{\frac{1}{3}} \cos(s^{\frac{4}{3}} - 8) ds$

$u = s^{\frac{4}{3}} - 8$   
 $\frac{du}{ds} = \frac{4}{3} s^{\frac{1}{3}}$   
 $du = \frac{4}{3} s^{\frac{1}{3}} ds$   
 $\frac{3}{4} du = s^{\frac{1}{3}} ds$

$= \frac{3}{4} \int \cos u du$   
 $= \frac{3}{4} \sin u + C$   
 $= \frac{3}{4} \sin(s^{\frac{4}{3}} - 8) + C$

(38)  $\int \frac{6 \cos t}{(2+\sin t)^2} dt$

$u = 2+\sin t$   
 $\frac{du}{dt} = \cos t$   
 $du = \cos t dt$   
 $6du = 6 \cos t dt$

$= 6 \int \frac{du}{u^2}$   
 $= 6 \int u^{-2} du$   
 $= 6(-1)u^{-1} + C$

$$= \left[ \frac{3}{4} \sin(s^{\frac{4}{3}} - 8) + C \right]$$

$$\begin{aligned} & - \int u \, du \\ &= 6(-1)u^{-1} + C \\ &= \left[ \frac{-6}{2 + \sin t} + C \right] \end{aligned}$$