6.1 Diff Eqs practice

Wednesday, February 25, 2015 6:50 AM

1) Decide which function from column 2 is a solution of the differential equation in column 1.

a) $\frac{dy}{dx} = 2y$ III

b) $y'' = -4y$ II

c) $\frac{dy}{dx} = -2y$ IV

d) $\frac{d^2y}{dx^2} = -y$ I

e) $y'' - 2y' + y = 0$ V

$2e^x + xe^x - 2(e^x + xe^x) + xe^x = 0$ ✓

I) $y = 2\sin(x)$

$\frac{dy}{dx} = 2\cos x \quad \frac{d^2y}{dy^2} = -2\sin x = -y$ d

II) $y = \sin(2x)$

$\frac{dy}{dx} = 2\cos 2x \quad \frac{d^2y}{dy^2} = -4\sin 2x = -4y$ b

III) $y = e^{2x}$

$\frac{dy}{dx} = 2e^{2x} = 2y$ a

IV) $y = e^{-2x}$

$\frac{dy}{dx} = -2e^{-2x} = -2y$ c

V) $y = xe^x$

$\frac{dy}{dx} = e^x + xe^x$

$\frac{d^2y}{dy^2} = e^x + e^x + xe^x$

2) A solution is given for each differential equation. Show that the function really is a solution of the differential equation and determine the particular solution that satisfies the given condition.

a. $\frac{dy}{dx} = \frac{1-2y}{x}$ $\quad y = \frac{k}{x^2} + \frac{1}{2}$ $\quad y(2) = 4$

$= \frac{1 - 2\left(\frac{k}{x^2} + \frac{1}{2}\right)}{x}$

$= \frac{1 - \frac{2k}{x^2} - 1}{x}$

$= \frac{-2k}{x^3}$ ✓

$\frac{dy}{dx} = \frac{-2k}{x^3}$ ✓

$4 = \frac{k}{2^2} + \frac{1}{2}$

$4 = \frac{k}{4} + \frac{1}{2}$

$\frac{7}{2} = \frac{k}{4}$

$14 = k$

$\boxed{y = \frac{14}{x^2} + \frac{1}{2}}$

b. $\frac{dy}{dt} = \sqrt{y}$ $\quad y = \frac{1}{4}(C+t)^2$ $\quad$ passes through (6,16)

$= \sqrt{\frac{1}{4}(C+t)^2}$

$= \frac{1}{2}(C+t)$ ✓

$\frac{dy}{dt} = \frac{1}{2}(C+t)$

$16 = \frac{1}{4}(C+6)^2$

$\sqrt{64} = \sqrt{(C+6)^2}$

$C+6 = 8 \qquad C+6 = -8$

$C = 2 \qquad C = -14$

$\boxed{y = \frac{1}{4}(2+t)^2}$