# 7.1 INTEGRAL AS NET CHANGE

**Displacement** VS. **Total Distance**

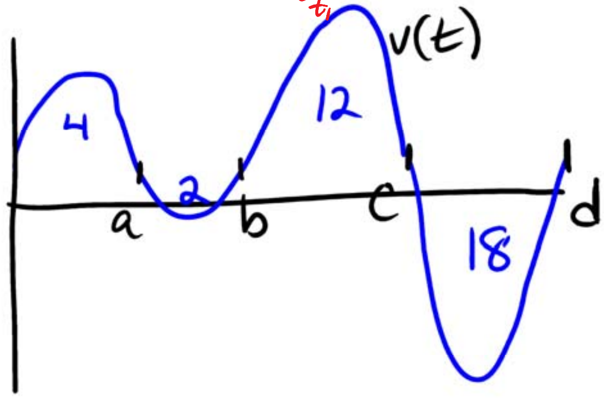
net change in position

$= \int_{t_1}^{t_2} v(t)\,dt$

distance = positive, so

$= \int_{t_1}^{t_2} |v(t)|\,dt$

A particle moves along the x-axis from time t = 0 to time $t = d$. Its initial position at $t = 0$ is s(0) = 5. The graph shows the particle's velocity $v(t)$. The numbers are the areas of the enclosed regions.




**Answer the following questions:**

1) When is the particle moving to the…

Right? when $v(t) > 0$
$t$: $(0,a)$ $(b,c)$

Left? when $v(t) < 0$
$(a,b), (c,d)$

When is it stopped? $v(t) = 0$
$t = a, b, c, d$

2) What is the particle's **displacement** and **total distance** from…

$t = 0$ to $t = a$? $\int_0^a v(t)\,dt = \boxed{4}$ | $\int_0^a |v(t)|\,dt = \boxed{4}$

$t = 0$ to $t = b$? $\int_0^a v(t)\,dt = 4 - 2 = \boxed{2}$ | $\int_0^b |v(t)|\,dt = \int_0^a v(t)\,dt - \int_a^b v(t)\,dt = 4 - (-2) = \boxed{6}$

$t = 0$ to $t = c$? $4 - 2 + 12 = \boxed{14}$ | $4 + 2 + 12 = \boxed{18}$

$t = 0$ to $t = d$? $4 - 2 + 12 - 18 = \boxed{-4}$ | $4 + 2 + 12 + 18 = \boxed{36}$

3) What is the particle's **position** at time…

a? $s(a) = s(0) + \int_0^a v(t)\,dt = 5 + 4 = \boxed{9}$

b? $s(b) = s(0) + \int_0^b v(t)\,dt = 5 + 2 = \boxed{7}$

c? $s(c) = s(0) + \int_0^c v(t)\,dt = 5 + 14 = \boxed{19}$

d? $s(d) = s(0) + \int_0^d v(t)\,dt = 5 + -4 = \boxed{1}$

For the following problems, the function v(t) is the velocity in cm/sec of a particle moving

For the following problems, the function v(t) is the velocity in cm/sec of a particle moving along the x-axis. Find the following:

a) When the particle is stopped, moving left, and moving right.
b) The displacement for the given time interval.
c) The particle's final position if s(0) = 5.
d) The total distance the particle traveled.

4) $v(t) = 4\cos 2t, \quad 0 \le t \le \pi$

Crit pts: $v(t) = 4\cos 2t = 0$

$\cos 2t = 0$

$2t = \cos^{-1}(0)$

$2t = \frac{\pi}{2} \quad 2t = \frac{3\pi}{2}$

$t = \frac{\pi}{4}, \ t = \frac{3\pi}{4}$

pos neg pos

$[0 \quad \frac{\pi}{4} \quad \frac{3\pi}{4} \quad \pi]$

ⓐ Right $[0, \frac{\pi}{4}) (\frac{3\pi}{4}, \pi]$ bc $v(t) > 0$

Left $(\frac{\pi}{4}, \frac{3\pi}{4})$ bc $v(t) < 0$

Stopped at $t = \frac{\pi}{4}, \frac{3\pi}{4}$ sec bc $v(t) = 0$

ⓑ disp $= \int_0^{\pi} 4\cos 2t\,dt = 4(\frac{1}{2})\sin 2t \Big|_0^{\pi}$

$= 2\sin 2t \Big|_0^{\pi} = 2\sin 2\pi - 2\sin 0$

$= \boxed{0}$

ⓒ $s(\pi) = s(0) + \int_0^{\pi} v(t)\,dt = 5 + 0 = \boxed{5}$

ⓓ dist traveled $= \int_0^{\pi} |v(t)|dt = \int_0^{\frac{\pi}{4}} v(t)dt - \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} v(t)dt + \int_{\frac{3\pi}{4}}^{\pi} v(t)dt$

$= 2\sin 2t \Big|_0^{\frac{\pi}{4}} - 2\sin 2t \Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}} + 2\sin 2t \Big|_{\frac{3\pi}{4}}^{\pi}$

$= (2\sin\frac{\pi}{2} - 2\sin 0) - (2\sin\frac{3\pi}{2} - 2\sin\frac{\pi}{2}) + (2\sin 2\pi - 2\sin\frac{3\pi}{2})$

$= (2-0) - (-2-2) + (0-(-2)) = 2 + 4 + 2 = \boxed{8}$

5) $v(t) = 49 - 9.8t, \quad 0 \le t \le 8$

$v(t) = 49 - 9.8t = 0$

$t = 5$

pos neg

$[0 \quad 5 \quad 8]$

ⓐ Right $[0, 5)$ bc $v(t) > 0$

Left $(5, 8]$ bc $v(t) < 0$

Stopped $t = 5$ bc $v(t) = 0$

ⓑ disp $= \int_0^8 v(t)dt = \int_0^8 (49 - 9.8t)dt = (49t - \frac{9.8}{2}t^2)\Big|_0^8 = 49(8) - 4.9(8)^2 - 0$

$= 392 - 313.6 = \boxed{78.4}$

ⓒ Final Postion $s(8) = s(0) + \int_0^8 v(t)dt = 5 + 78.4 = \boxed{83.4}$

ⓓ Total dist traveled $= \int_0^8 |v(t)|dt = \int_0^5 v(t)dt - \int_5^8 v(t)dt = (49t - 4.9t^2)\Big|_0^5 - (49t - 4.9t^2)\Big|_5^8$

$= ((49(5) - 4.9(5)^2) - 0) - ((49(8) - 4.9(8)^2) - (49(5) - 4.9(5)^2))$

$(122.5) - ((78.4) - 122.5) = \boxed{166.6 \text{ cm}}$