Tuesday, January 24, 2017

- ✓ **HW Questions**
- ✓ **6.4 - The FUNdamental Theorem of Calculus**

FTC, It's Fundamental!

6.4 Fundamental Theorem of (Integral) Calculus

2 Parts

FTC: Antiderivative Part

If f is cont on $[a,b]$ then:

$F(x) = \int_a^x f(t)\,dt$ has a derivative at every point in $[a,b]$ and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

* f is the derivative of a function: $\int_a^x f(t)\,dt = F(x)$
* every continuous function has an antiderivative

* Differentiation and Integration are Inverses

Ex: ① $\frac{d}{dx}\int_0^x \sin^2 t\,dt = \sin^2 x$

Why? work this out:

② $\frac{d}{dx}\int_7^x \sin a\,da = \frac{d}{dx}\left[-\cos a\right]_7^x$

$= \frac{d}{dx}\left[-\cos x + \cos 7\right]$

$= \sin x + 0$

$= \boxed{\sin x}$

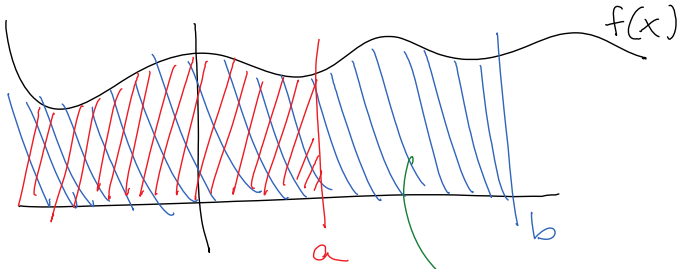
③ $\frac{d}{dx}\int_3^x (3t^2 - t + 2)\,dt = 3x^2 - x + 2$

④ $\frac{d}{dx}\int_x^3 (4e^3 + 2e^2 - e + 5)\,de =$

$\frac{d}{dx}\left[-\int_3^x 4e^3 + 2e^2 - e + 5)\,de = -4x^3 - 2x^2 + x - 5\right.$

FTC Part 2: Evaluation Part

$$\int_a^b f(x)\,dx = F(b) - F(a)$$



F(b) = accumulated area up to b

F(a) = accumulated area up to a

F(b) − F(a) = area between a and b