

Suppose f and g are continuous functions and that

$$\int_{-3}^2 g(x) dx = -4 \quad \int_0^2 g(x) dx = 1 \quad \int_{-3}^2 f(x) dx = 4$$

Find each integral.

a. $\int_{-3}^0 g(x) dx = \int_{-3}^2 g(x) dx - \int_0^2 g(x) dx$
 $= -4 - 1$
 $= \boxed{-5}$

b. $\int_{-3}^2 \left[0.5f(x) - \frac{g(x)}{5} \right] dx =$
 $= 0.5(4) - \left(\frac{-4}{5} \right)$
 $= 2 + \frac{4}{5} = \boxed{2\frac{4}{5}}$

Find the average value of the function on the interval. (For a) use the antiderivative; for b) use geometry to evaluate the integral.)

a. $f(x) = 3x^2 + 1$ $[-1, 2]$

$$\begin{aligned} \text{av}(f) &= \frac{1}{2 - (-1)} \int_{-1}^2 (3x^2 + 1) dx \\ &= \frac{1}{3} (x^3 + x) \Big|_{-1}^2 \\ &= \frac{1}{3} ((8+2) - (-1+1)) \\ &= \frac{1}{3} (10+0) \end{aligned}$$

$\boxed{\text{av}(f) = 4}$

where does $f(x) = \text{av}(f)$?

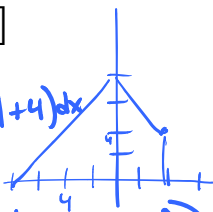
$$3x^2 + 1 = 4$$

$$3x^2 = 3$$

$$x^2 = 1$$

$\boxed{x = \pm 1}$ (both are in interval)

b. $g(x) = -|x| + 4$ $[-4, 2]$

$$\begin{aligned} \text{av}(g) &= \frac{1}{2 - (-4)} \int_{-4}^2 (-|x| + 4) dx \\ &= \frac{1}{6} \left(\frac{1}{2}(4)(4) + \frac{1}{2}(4+2)(2) \right) \\ &= \frac{1}{6} (8 + 6) \\ &= \frac{14}{6} \\ &= \boxed{\frac{7}{3}} \end{aligned}$$


where does $g(x) = \text{av}(g)$?

$$-|x| + 4 = \frac{7}{3}$$

$$-|x| = -\frac{5}{3}$$

$$|x| = \frac{5}{3}$$

$\boxed{x = \frac{5}{3}, -\frac{5}{3}}$ both in interval